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Telefunctioning: An Approach to Telerobotic Manipulations

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ABSTRACT

This article introduces three concepts: 1) "telefunctioning", 2) a control method for achieving telefunctioning, and 3) an approach to the analysis of human-robot interaction when telefunctioning governs the system behavior. Telefunctioning facilitates maneuvering of loads by creating a perpetual sense of the load dynamics for the operator. We define telefunctioning as a robotic manipulation method in which the dynamic behaviors of the slave robot and the master robot are functions of each other; these functions are the designer's choice and depend on the application. In a subclass of telefunctioning called telepresence, all of the relationships between the master and the slave are specified as "unity" so that all of the master and slave variables (e.g., position, velocity) are dynamically equal. To create telefunctioning, we arrive at a minimum number of functions relating the robots' variables. We then develop a control architecture which guarantees that the defined functions govern the dynamic behavior of the system. The stability of the closed-loop system (master robot, slave robot, human, and the load being manipulated) is analyzed and sufficient conditions for stability are derived.

1. DEFINITION OF TELEFUNCTIONING

A telerobotic system consists of two robots: the "master" which is maneuvered by a human, and the "slave" which performs a task at a location remote from the master. The master robot is not connected mechanically to the slave robot. Figure 1 shows a telerobotic system where a human is pushing against the master and the slave is pushing against an environment[2]. "Telepresence" denotes a dynamic behavior in which the environmental effects experienced by the slave are transferred through the master to the human without alteration; therefore, the human feels that he[3] is "there" without "being" there [2, 3, 9,13]. In the following three examples, we discuss the concept of "telefunctioning" and how it differs from "telepresence".

Example 1

Suppose a telerobotic system is used to manipulate an object through a completely arbitrary trajectory. We may want a system dynamic behavior in which the human feels scaled-down values of the forces that the slave experiences when maneuvering the object. Therefore, we want to design a system controller for which the ratio of the forces on the slave to the forces on the master equals a number greater than unity. If f_m[4] and f_s represents the forces on the master and on the slave, then $f_s = -\alpha f_m$ where α is a scalar greater than unity. (The negative sign, originating from the convention used in Figure 1, implies the opposite directions of f_m and f_s.)

If the object being manipulated is a pneumatic jackhammer, we may want to both filter and decrease the jackhammer forces. Then, the human feels only the low-frequency, scaled-down components of the forces that the slave experiences. This requires a low-pass filter such that $f_s=-\alpha f_m$ where $1/\alpha$ is a low-pass filter transfer function.

In this example of telefunctioning, the slave forces are functions of the master forces so the human senses forces different from those which the slave senses: in general the slave and master forces are not equal as they would be in the case of telepresence.

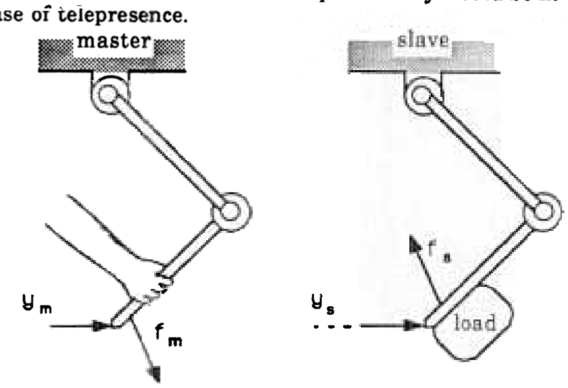

Figure 1: In a telerobotic system, a human constrains the motion of the master robot while an environment constrains the motion of the slave robot.

Example 2

We may want a dynamic behavior for the telerobotic system in which the human, who is maneuvering a rigid body, feels the forces as being those of maneuvering a light single-point mass. This dynamic behavior masks the cross-coupled forces associated with maneuvering a rigid body; the human feels only the forces associated with the acceleration of a single-point mass. This behavior is desirable because cross-coupled forces contribute to the difficulty of maneuvering a rigid body. In contrast to this telefunctioning example, when telepresence governs the system behavior, the forces on the master robot and slave robot are equal, and the human would feel all of the forces, including the cross-coupled forces, associated with maneuvering a rigid body.

For the example above, the relationship between the forces on the master robot and the forces on the slave robot cannot be given explicitly by an equation. Later, we will develop a mathematical tool to frame the design specifications needed in this situation.

Example 3

In another example, we may want a behavior for the telerobotic system in which the slave robot position (not force, as in example 1) equals a scaled-down value of the master robot position. In other words, if y_m and y_s are the positions of the master robot and the slave robot, then $y_s=\beta y_m$ where β

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[2] In this article, the word *environment* represents any object being manipulated or pushed by the slave robot.

[3] The pronouns "he", "his", and "him" are not meant to be gender-specific.

[4] The subscript "m" signifies the master and "s" signifies the slave. Unless otherwise noted, all variables are defined in Laplace domain. The Laplace argument for all functions are omitted.

is smaller than unity. This behavior is useful when great precision is required in the slave maneuver; a few centimeters of master motion correspond to a few microns of slave motion. This would have applications in micro-surgery. In contrast to this telefunctioning example, when telepresence governs the system behavior, y_m and y_s are equal.

In each of the above examples, one relationship between the master robot and slave robot variables is the performance specification for telefunctioning. But, several independent relationships can specify a particular type of telefunctioning. Here, we mathematically frame telefunctioning in terms of relationships which are independent of the chosen control techniques. Without formal proof[5], we state that, for linear systems, only three independent relationships can be specified among the four variables: y_m, y_s, f_m, and f_s. One possible set of relationships is:

$$y_s = A_y\, y_m \tag{1}$$

$$f_s = A_f\, f_m \tag{2}$$

$$f_s = Z_s\, y_s \tag{3}$$

A_y, A_f, and Z_s are transfer functions. A_y and A_f represent the relationships between the positions and forces while Z_s is the slave port impedance. Note that, once the above three relationships are specified, no other independent relationships can be specified. Figure 2 shows the variables and their relationships graphically where the thick lines represent the specified relationships (equations 1, 2, and 3) and the thin lines portray other dependent relationships.

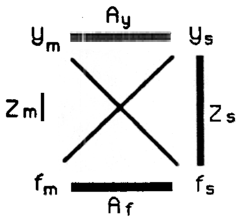

Figure 2: If three relationships (thick lines) are specified among the four variables, then any other relationships (thin lines) will depend on the specified relationships.

Employing equations 1, 2, and 3, the dynamic behavior of the system in example 1 can be expressed mathematically by the following three equations.

$$A_f = -\alpha \tag{4}$$

$$A_y = 1 \tag{5}$$

$$Z_s = \text{arbitrary} \tag{6}$$

A_f in equation 4 is the force amplification, and A_y in equation 5 states the equality of the master robot and slave robot positions. Equation 6 shows that designers can freely choose the slave port impedance, Z_s. Note that, once equations 4, 5, and 6 are specified, no other equations can be specified for the system.

Applying equations 1, 2, and 3 to example 2, the design specifications for telefunctioning become:

$$A_f = -1 \tag{7}$$

$$A_y = 1 \tag{8}$$

$$Z_s = ms^2 \tag{9}$$

where m represents the desired mass and s is the Laplace operator. Substituting y_s and f_s from equations 1 and 2 into equation 3 (while incorporating equations 7, 8, and 9) yields a relationship between y_m and f_m such that $f_m = ms^2\, y_m$. This proves that the human, when maneuvering a rigid body, would feel the forces due to maneuvering a single-point mass m.

A set of performance specifications for telefunctioning (e.g., equations 1, 2, and 3) does not assure system stability but does let designers express what they wish to have happen during a maneuver. If A_f and A_y are specified as "unity", then telepresence is seen to be a subclass of telefunctioning in which all the variables (e.g., position, velocity, force) of the master robot and the slave robot are dynamically equal. But, in telefunctioning, the dynamic behaviors of the slave robot and master robot are functions of each other; these functions are the designer's choice and depend on the application. Note that, although the design specifications described above are independent of the control method used, we will propose a new and practical control architecture in the next section to achieve these specifications and create telefunctioning.

2. THE CONTROL ARCHITECTURE

The control architecture to create telefunctioning has the following properties:

1. It lets designers handle the robustness of the master robot and the slave robot without getting involved in the dynamics of the human, the dynamics of the object being manipulated by the slave, or the communication time delay [1]. In other words, the designers can minimize the sensitivity of the master robot and the slave robot to uncertain dynamic modeling of each robot independent of any other variables.
2. This control architecture is the most general extension of the previous telerobotic control architectures (described in [3]) and allows a variety of performance specifications. We will show how conveniently the design specifications, described in Section 1, will be mapped onto the variables of the proposed control architecture.
3. The human wearing the master robot is in physical contact with the machine, so power transfer is unavoidable and information signals from the human help to control both master robot and slave robot. The proposed control architecture conveniently depicts these two paths of human-machine interaction.

To understand the control law, we use linear control theory for a single-degree-of-freedom telerobotic system; thus, we can employ the rich concepts of linear control theory. Understanding the proposed closed-loop control approach requires understanding the dynamic behaviors of the master and slave robots, the human arm, and the environment, as discussed in the following sub-sections.

Dynamic Behaviors of the Master Robot and the Slave Robot

It is assumed that both the master robot and the slave robot primarily have independent closed-loop position controllers. The use of these primary stabilizing compensators[6] in both the master and the slave is motivated by the following:

1. For the safety of the human, the master must remain stable when not worn by the human. A closed-loop position controller keeps the master stationary when not worn by the human.
2. This controller also minimizes the effects of frictional forces in the joints and the transmission mechanism, thus attenuating the sensitivity of each robot to uncertain forces.

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[5] One can show, by Bond Graph Theory [11], that the system causality will be violated if more than three relationships are specified among the four variables: y_m, y_s, f_m, and f_s. For the sake of brevity, the proof is not given here.

[6] Hereafter, the words *primary stabilizing compensators* refer to two closed-loop position controllers that stabilize the master robot and the slave robot. These controllers also eliminate the effect of friction forces in the robots' joints.

3. The design of the primary stabilizing compensator lets the designers deal with the robustness of the master robot and the slave robot without getting involved in the dynamics of the human, the dynamics of the object being manipulated by the slave, or the communication time delay. A variety of robust control methods can be used to stabilize the master and slave robots independently. (Refer to [12, 14] for two well-established robotic trajectory control techniques.)

Only the master dynamic behavior is derived here; the derivation of the dynamic behavior of the slave robot is similar to that of the master robot. The master robot position, y_m, results from two inputs: the electronic command to the primary controller of the master robot and the forces imposed on the master robot. The transfer function G_m is defined as the primary closed-loop system with the electronic command u_m as the input and the master position, y_m, as the output. The master robot "sensitivity" transfer function, S_m, maps the force imposed on the master robot, f_m, onto the master position, y_m. (S_m is the reciprocal of the robot stiffness.) Equation 10 represents the master robot dynamic behavior in its most general form.

$$y_m = G_m u_m + S_m f_m \tag{10}$$

Since the master robot is in contact with only the human, f_m represents forces from only the human. The motion of the master robot has a small response to the human forces, f_m, if the magnitude of S_m is small. Use of a high-gain closed-loop positioning system as the primary controller or use of an actuator with a large gear ratio yields a small S_m. [4, 6]

Similarly, the dynamic behavior of the slave robot can be defined by equation 11.

$$y_s = G_s u_s + S_s f_s \tag{11}$$

f_s is the force imposed on the slave endpoint and u_s is the input command to the primary controller of the slave drive system. G_s and S_s are similar to G_m and S_m, and represent the effects of u_s and f_s. We will soon learn that the forces imposed on the slave robot, f_s, are from the environment.

Dynamic Behavior of the Human Arm

The human arm dynamic behavior is modeled as a functional relationship between a set of inputs and a set of outputs. Therefore, the internal structures of the model components are not of concern: the particular dynamics of nerve conduction, muscle contraction, and central nervous system processing are implicitly accounted for in constructing the dynamic model of the human arm. (Refer to [8, 10, 15] for a thorough review on various dynamic models of the human arm.)

The human arm is modeled as a non-ideal force control system in which the force imposed by the human arm on the master robot is the result of two inputs. The first input, u_h, is issued by the human central nervous system; it is assumed that the specified form of u_h is not known other than it is human thought deciding to impose a force on the master robot. The second input is the position of the master robot. Thus, the master robot motion can be thought of as a position disturbance occurring on the force-controlled human arm. If the master robot is stationary, the force imposed on the master robot is only a function of commands from the central nervous system. If the master robot moves, the force imposed on the master robot is a function not only of the central nervous system commands but also of the master robot position, and the amount of force imposed on the master robot will be different from u_h. The transfer function S_h maps the master robot position, y_m, into the force imposed on the master robot, f_m.

$$f_m = u_h - S_h y_m \tag{12}$$

S_h, the human arm "sensitivity" function (or impedance [5, 6]), is the disturbance rejection property of the human arm. If the gain of S_h is small, the master robot motion has a small effect on the imposed forces, f_m.

Dynamic Behavior of the Environment

Telerobotic systems are used for manipulating objects or imposing forces on objects. Defining E as a transfer function representing the environment dynamics and f_{ext} as the equivalent of all the external forces imposed on the environment, equation 13 provides a general expression for the force imposed on the slave robot in the linear domain[7].

$$f_s = - E y_s + f_{ext} \tag{13}$$

If the slave robot is employed for pushing a spring and damper, E is a transfer function such that $E(s) = (K + Cs)$ and $f_{ext} = 0$ where K, C, y_s, and s are the stiffness, damping, slave position, and Laplace operator. In another example, if the slave robot is employed to maneuver a mass, then the dynamics of the object being manipulated is represented by $E(s) = m_o s^2$ where m_o is the mass of the object.

The dynamic behavior of the telerobotic system, the human arm, and the environment is represented by the block diagram of Figure 3 where equations 10, 11, 12, and 13 are dynamic models of the master robot, the slave robot, the human arm, and the environment. In the diagram, H is the control feedback operating on the contact forces. Note that there is no cross-feedback between the positions; only the forces are measured for feedback. This is a fundamental difference between this control law and previous control methods. (See [3] for a summary of previous telerobotic control methods).

In Figure 3, if u_s, u_m, u_h, and f_{ext} are zero (i.e., the inputs to the master robot and the slave robot are zero, the human has no intention of moving the master robot, and no other forces are imposed on the slave) and H_{11} and H_{21} are chosen to be zero, the interaction force between the human and the master will be zero. If the human decides to move his hand (i.e., u_h becomes a nonzero value) and u_m, f_{ext}, H_{11}, and H_{21} are still zero, a small master motion will develop as a result of the interaction force between the master and the human. The master motion will be trivial if S_m has a small gain, even though the interaction force may not be small. In other words, the human arm may not have the strength to overcome the master primary control loop.

The interaction force f_m is measured and filtered by compensator H_{11} and then used as an input to the master primary controller. At this point, there is no restriction placed on the structure and size of H_{11}. The interaction force f_m is also used to drive the slave robot after passing through the compensator H_{21}. If $H_{11} = H_{21}$, the master and slave motion are the same. Note that the mapping $G_m H_{11}$ acts in parallel to S_m and thus increases the apparent sensitivity of the master robot. Figure 3 suggests choosing a large gain for H_{11} to increase the apparent sensitivity of the master robot.

Similarly, compensator H_{22} is chosen to generate compliancy in the slave robot in response to the forces, f_s, imposed on the slave robot endpoint [4]. The interaction

[7] One can think of f_{ext} as the equivalent of all the forces on the slave robot endpoint which do not depend on y_s and other system variables. One example for f_{ext} can be observed when a second human is holding and maneuvering the slave endpoint; the force imposed on the slave endpoint by the human represents f_{ext}. In this article it is assumed that $f_{ext} = 0$.

force f_s also affects the master robot as a force reflection after passing through the compensator H_{12}.

Our goal is to find the H transfer function matrix such that the satisfaction of equations 1, 2, and 3 is guaranteed for the system. Designers do not have complete freedom in choosing the structure and magnitude of H: the closed-loop system must remain stable for any chosen values of H. This controller creates a desired behavior for the master and slave based on the human arm and environment models generated by the computer. The output of this controller is then fed to both the master and slave drive systems. The master robot also interacts physically with the human; the master motion, then, is partially due to the transfer of human power via S_m (shown by double lines in Figure 3) and partially due to the command generated by the computer via H. The slave robot interacts physically with the environment; its motion, then, is partially due to the transfer of power from the environment through S_s (shown by double lines) and partially due to the command generated by the computer via H. The command to the slave robot must be such that the total slave maneuver becomes a desired maneuver that the person could not achieve alone [5].

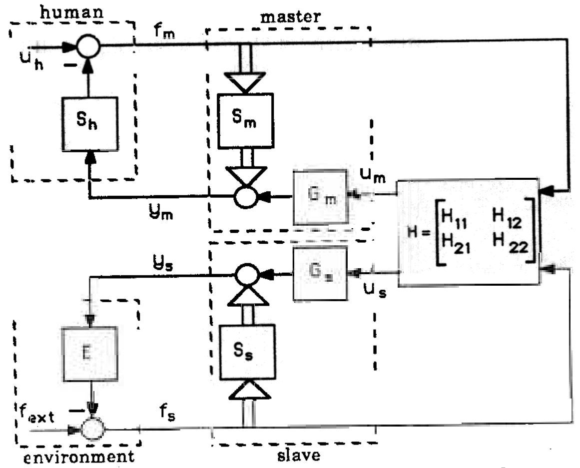

Figure 3: The major elements of the proposed control architecture which creates telefunctioning.

3. STABILITY ANALYSIS

This section describes the design method for the controller which creates telefunctioning. The design objective is to select H such that the achievement of the design specifications in equations 1, 2, and 3 is guaranteed. Here we describe a simple control method when a linear system governs the dynamic behavior of the system; this will lead the reader to the general solution that we propose. Inspection of the block diagram of Figure 3 results in the following equations:

$$A_y = \frac{y_s}{y_m} = \frac{P_{21}}{P_{11} + \delta P\, E} \qquad (14)$$

$$A_f = \frac{f_s}{f_m} = -\frac{P_{21} E}{1 + P_{22} E} \qquad (15)$$

$$Z_m = \frac{f_m}{y_m} = \frac{1 + P_{22} E}{P_{11} + \delta P\, E} \quad \text{when } u_h = 0 \qquad (16)$$

$$Z_s = \frac{f_s}{y_s} = \frac{1 + P_{11} S_h}{P_{22} + \delta P\, S_h} \qquad (17)$$

where:

$$P_{11} = G_m H_{11} + S_m \qquad (18)$$

$$P_{12} = G_m H_{12} \qquad (19)$$

$$P_{21} = G_s H_{21} \qquad (20)$$

$$P_{22} = G_s H_{22} + S_s \qquad (21)$$

$$\delta P = P_{11} P_{22} - P_{12} P_{21} \qquad (22)$$

By inspection, it can be observed that equations 14, 15, and 16 are not independent and that they satisfy the following equation:

$$\frac{A_y}{A_f} = -\frac{Z_m}{E} \qquad (23)$$

Thus, once A_y and A_f are specified (via equations 1 and 2), the designer cannot choose Z_m arbitrarily: Z_m must be derived from equation 23. Therefore, the design specifications must include an arbitrary choice for Z_s and two choices from among the three variables A_y, A_f, and Z_m. (This confirms that only three relationships are necessary to sufficiently describe the system behavior.) Here we choose A_y, A_f, and Z_s as the design specifications[8] and solve three equations 14, 15, and 17 to calculate four unknowns: P_{11}, P_{12}, P_{21}, and P_{22}. Once these are calculated, the members of the H matrix can be found from equations 18 through 21. Since equations 14, 15, and 17 contain four unknowns, arbitrary assignment of P_{11} leads to the following solutions for P_{12}, P_{21}, and P_{22}.

$$P_{12} = -\frac{A_f + A_y E P_{11}}{A_f A_y E} \qquad (24)$$

$$P_{21} = -\frac{A_f A_y (1 + S_h P_{11})(E + Z_s)}{Z_s (A_y E - A_f S_h)} \qquad (25)$$

$$P_{22} = \frac{A_y E^2 (1 + S_h P_{11}) + S_h Z_s (A_f + A_y E P_{11})}{E Z_s (A_y E - A_f S_h)} \qquad (26)$$

Once P_{12}, P_{21}, and P_{22} are found from equations 24, 25, and 26, members of H can be found from equations 18 through 21. In the above method, designers have complete freedom to select Z_s, A_y, and A_f. However, the stability of the closed-loop system in Figure 3 is not guaranteed for all possible values of Z_s, A_y, and A_f.

Using the Nyquist stability criterion (Appendix A), it can be found that the following two conditions are needed to guarantee the stability of the closed-loop system shown in Figure 3:

$$|P_{11}| < \frac{1}{|S_h|} \qquad \text{for all } \omega \in [0, \infty) \qquad (27)$$

$$\left| \frac{1 + P_{11} S_h}{P_{22} + \delta P\, S_h} \right| > |E| \qquad \text{for all } \omega \in [0, \infty) \qquad (28)$$

The left-hand side of equation 28 equals Z_s (defined in equation 17). The stability conditions in inequalities 27 and 28 can be satisfied easily because the left-hand sides of the inequalities are the designers' choice . We must choose Z_s to be larger than E and P_{11} to be smaller than $1/S_h$ in the sense of magnitude. This presents an interesting property of the proposed control law: the stability criteria (inequalities 27 and 28) do not limit the designer in choosing the design specifications described by A_f and A_y; the limitation only restricts the designers in choosing Z_s. To summarize, the designers can choose three functions to describe the system behavior A_f, A_y, and Z_s; while there is no restriction on the choice of A_f and A_y, Z_s (slave impedance) must be larger than E (inequality 28).

4. EXAMPLE

Consider a one-degree-of-freedom telerobotic system shown in Figure 4. The master robot is a link powered by a DC motor. The human holds the handle on the link to maneuver the master arm. A force sensor between the handle and the link measures the human contact force. The slave robot is also a link powered by a DC motor. A mass, simulating a load, is attached to the slave link. A force sensor located between the slave robot and the load measures the load force. Two independent primary stabilizing controllers for the master robot and for the slave robot have

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[8] One can choose A_y, Z_m, and Z_s, or A_f, Z_m, and Z_s as the set of design specifications.

been designed to yield the widest bandwidth for the closed-loop transfer functions, G_m and G_s, while guaranteeing the stability of each system in the presence of bounded unmodeled dynamics. The dominant dynamics for G_m, G_s, S_m, and S_s representing the closed loop positioning system are given by equations 29 through 32. The development of the position controllers for both robots has been omitted for brevity.

$$G_m = \frac{0.95}{0.1\,S+1} \quad \text{rad/rad} \tag{29}$$

$$G_s = \frac{0.9}{0.05\,S+1} \quad \text{rad/rad} \tag{30}$$

$$S_m = \frac{0.03}{0.1\,S+1} \quad \text{rad/lbf} \tag{31}$$

$$S_s = \frac{0.05}{0.05\,S+1} \quad \text{rad/lbf} \tag{32}$$

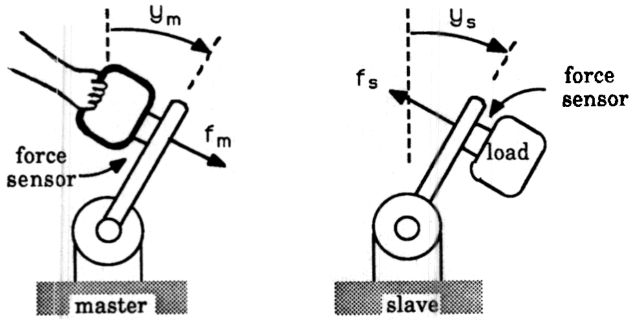

Figure 4: A One-Degree-of-Freedom Telerobotic System

Note that the master robot has a bandwidth of about 10 rad/sec while the slave's bandwidth is about 20 rad/sec. Since G_m and G_s transform position commands to actual robot positions, their units are rad/rad. Transfer functions S_m and S_s represent the sensitivity of each system to forces and their units are rad/lbf.

Based on several experiments, at various frequencies, an estimate for the human's arm sensitivity is given in [5, 6]. Equation 34 represents an approximation of the human arm dynamics in the neighborhood of the Figure 4 configuration when the master robot deviation from the vertical line is small.

$$S_h = 2.5\,(s+1)^2 \quad \text{lbf/rad} \tag{33}$$

The slave robot is employed to maneuver a mass with an inertia such that:

$$E = 10\,s^2 \quad \text{lbf/rad} \tag{34}$$

The design objective is to design the H matrix such that the human feels 1/5 of the force imposed on the slave while the master and slave positions are equal: $A_f=5$ and $A_y=1$. No specification for Z_s is given; therefore, Z_s is chosen to be $11s^2$ satisfying the second stability condition in inequality 28. P_{11} is chosen to be $\frac{1}{5(1+s)^2}$ satisfying the first stability condition in equality 27. Substituting the above values into equations 24, 25, and 26 results in transfer functions for P_{12}, P_{21} and P_{22} and the H matrix from equations 18-21.

$$H_{11} = \frac{-1.5\,s^2 - 2s + 8.5}{47.5\,(s+1)^2} \tag{35}$$

$$H_{12} = -\frac{(s+10)(7s^2+10s+5)}{475\,s^2(s+1)^2} \tag{36}$$

$$H_{21} = 0.3182\,\frac{(s+20)}{(s+0.5279)(s+9.4721)} \tag{37}$$

$$H_{22} = -\frac{(s+3.12)(s+7.43)(s^2+0.687s+0.43)}{18s^2(s+0.5279)(s+9.4721)} \tag{38}$$

The system was maneuvered irregularly by an operator for 10 seconds. The initial positions of the master robot differs from the initial position of the slave robot. Figure 5 shows the master and slave positions where y_m and y_s approach each other after the initial transition period. Figure 6 depicts the master and slave forces for the same maneuver where they are initially equal. After the initial transition period, the master force, f_m, becomes five times less than the slave force, f_s ($-f_s$ is plotted in this Figure). Figure 7 shows $-f_s$ vs f_m where the slope of 5 confirms that the operator feels 20% of the slave force.

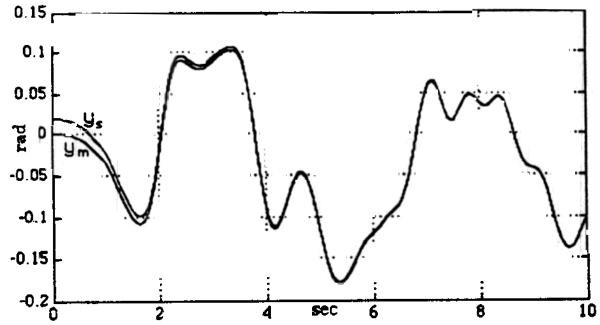

Figure 5:Master and Slave Positions

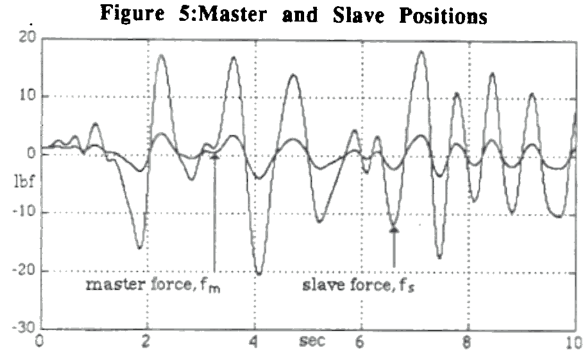

Figure 6: The slave force is five times larger than the master force.

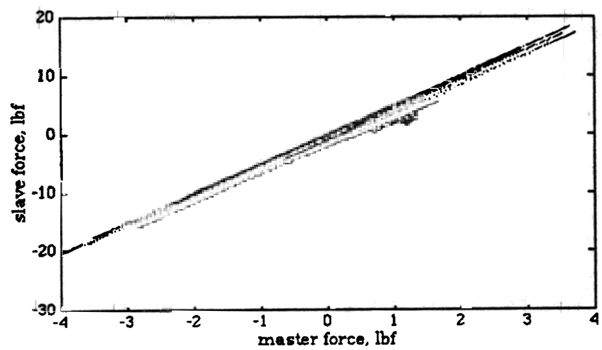

Figure 7: The Slave Force vs the Master Force

SUMMARY AND CONCLUSION

This article introduces robotic "telefunctioning. Telefunctioning facilitates maneuvering of loads by creating a perpetual sense of the load dynamics for the operator. We have defined minimum number of functions to frame the telefunctioning specifications. The stability of the system (master robot, slave robot, human, and the load being manipulated) is analyzed and sufficient conditions for stability are derived.

APPENDIX A

A sufficient condition for stability of the closed-loop system of Figure 3 is developed by the Nyquist Theorem [7]. This sufficient condition results in a class of compensators, H which guarantee the stability of the closed-loop system in Figure 3. Note that the stability condition derived in this section does not give any indication of system performance, but only ensures a stable system. After some manipulations, the block diagram of Figure 3 can be represented by the diagram of Figure A1.

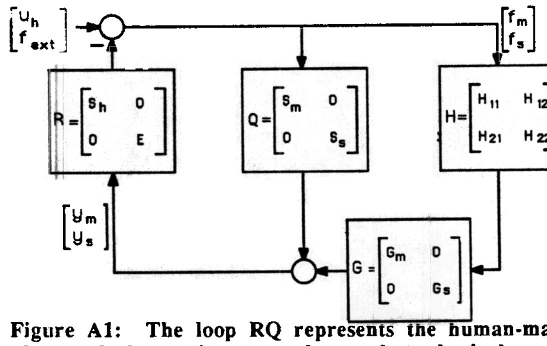

Figure A1: The loop RQ represents the human-master robot and the environment-slave robot physical contact.

An assumption is made that the system in Figure A1 is stable when $H = 0$. The plan is to determine how robust the system is when the term H is added to the feedback loop. Note that there are two elements in the feedback loop: RQ represents the natural feedback loops which occur as a result of the interaction between the human and master robot and between the environment and the slave robot while RGH represents the controlled feedback loop. If the controllers in the feedback loop are eliminated by setting $H = 0$, the system reduces to the case where the human wears the master robot and the slave is in contact with the environment, but command inputs to the primary controller of both robots are zero. The goal is to obtain a sufficient stability condition when H is added to the system. To achieve this, the Nyquist criterion is used. The following assumptions are made:

1. The closed-loop system in Figure A1 is stable when $H = 0$. This assumption states that the system of human and master robot taken as a whole and the system of environment and slave robot taken as a whole remain stable when no feedback compensator, H, is used in the system.
2. H is populated with stable linear transfer functions. Therefore, the loop transfer function, RQ, is the same number of right half-plane poles as $(RQ + RGH)$. For convenience in stability analysis, we assume $A=RQ$ and $B=RQ+RGH$

According to the Nyquist criterion, the system shown in Figure A1 remains stable as long as the number of anti-clockwise encirclements of det $(I+B)$ around the origin of the s-plane is equal to the number of unstable poles of the loop transfer function, B. By assumptions 1 and 2, B and A have the same number of unstable poles. Assuming that the system is stable when $H = 0$, the number of encirclements of the origin by det $(I+A)$ is equal to the number of unstable poles in A. When compensator H is added to the system, the number of encirclements of the origin by det $(I+B)$ must be equal to the number of unstable poles in B in order to guarantee closed-loop stability. Because of the assumption that the number of unstable poles in B and A is identical, det $(I+B)$ must have exactly the same number of encirclements of the origin as det $(I+A)$. In order to guarantee equal encirclements by det $(I+A)$ and det $(I+B)$, insurance is needed so that det $(I+B)$ does not pass through the origin of the s-plane for all frequencies.

$$\det(I + RQ + RGH) \neq 0 \quad \text{for all } \omega \in [0, \infty) \qquad (A1)$$

Substituting R, Q, G, and H from Figure A1 into equation A1 and calculating the determinant results in:

$$S_h E\,\delta P + P_{22} E + S_h P_{11} + 1 \neq 0 \text{ for all } \omega \in [0, \infty) \qquad (A2)$$

$$\text{if } S_h P_{11} + 1 \neq 0 \quad \text{for all } \omega \in [0, \infty) \qquad (A3)$$

Then, dividing A2 by A3 results in:

$$1 + \frac{(S_h\,\delta P + P_{22})E}{S_h P_{11} + 1} \neq 0 \quad \text{for all } \omega \in [0, \infty) \qquad (A4)$$

To ensure the truth of A4, one must guarantee that:

$$\left| \frac{(S_h\,\delta P + P_{22})E}{S_h P_{11} + 1} \right| < 1 \quad \text{for all } \omega \in [0, \infty) \qquad (A5)$$

Therefore, to ensure the stability of the system in Figure A1, inequalities A3 and A5 must be guaranteed; these inequalities are restated as follows:

$$\left| \frac{1 + P_{11} S_h}{P_{22} + \delta P\, S_h} \right| > |E| \quad \text{for all } \omega \in [0, \infty) \qquad (A6)$$

$$|P_{11}| < \frac{1}{|S_h|} \quad \text{for all } \omega \in [0, \infty) \qquad (A7)$$

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