A Controller Design Framework for Telerobotic Systems

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Abstract—This paper presents a framework for designing a telerobotic system controller. This controller is designed so the dynamic behaviors of the master robot and the slave robot are functions of each other. This paper first describes these functions, which the designer chooses based upon the application, and then proposes a control architecture to achieve these functions. To guarantee that the specified functions and proposed architecture govern the system behavior, $H_{\infty}$ control theory and model reduction techniques are used. Several experiments were conducted to verify the theoretical derivations. This control method is unique, because it does not require any transfer of either position or velocity information between the master robot and the slave robot; it only requires the transfer of forces. Although this property leads to a wider communication bandwidth between the master and slave robots, the entire system may still suffer from a positional error buildup between the master robot and slave robot.

I. TELEROBOTIC PERFORMANCE SPECIFICATIONS

A TELEROBOTIC system consists of a master robot and a slave robot which are not connected to each other mechanically. Fig. 1 shows a telerobotic system where a human is pushing against a master and a slave is pushing against an environment. "Telepresence" denotes a dynamic behavior in which the environmental effects experienced by the slave are transferred through the master to the human without alteration; therefore, the human feels that she/he is "there" without "being" there [3], [7], [8], [20]. The three examples below describe a few of the methods that may be used to specify the desired telerobotic performances. These examples are followed by a formal expression of the telerobotic design specifications.

Example 1: Suppose a telerobotic system is used to manipulate an object through a completely arbitrary trajectory. The goal may be a dynamic behavior for the telerobotic system in which the human senses scaled-down values of the forces that the slave senses when manipulating the object. Therefore, a system controller must be designed so that the ratio of the forces on the slave robot to the forces on the master robot equals a number greater than unity. If $f_s$ and $f_m$ represent the forces on the slave and on the master, then $f_s = -\alpha f_m$, where $\alpha$ is a scalar greater than unity. (The negative sign is based on the convention used in Fig. 1.)

Example 2: If the object being manipulated is a pneumatic jackhammer, the goal may be to both filter and decrease the jackhammer forces. Then, the human senses only the low-frequency, scaled-down components of the forces that the slave senses. This requires a low-pass filter such that $f_s = -\beta f_m$ where $1/\beta$ is a low-pass filter transfer function. As in Example 1, the slave forces are functions of the master forces so the human senses forces different from those which the slave senses.

Example 3: In maneuvers over an arbitrary trajectory, the goal may be a behavior for the telerobotic system in which the slave robot position (not force, as in Examples 1 and 2) equals a scaled-down value of the master robot position. In other words, if $y_s$ and $y_m$ are the positions of the slave robot and the master robot, then $y_s = \gamma y_m$, where $\gamma$ is a scalar smaller than unity. This behavior is useful when great precision is required in the slave maneuver; a few centimeters of master motion correspond to a few microns of slave motion. This would have applications in microsurgery. In each of the above examples, one relationship between the master robot and slave robot variables is chosen as the performance specification for telerobotics. However, several independent relationships might be chosen to specify a particular type of telerobotic behavior. In general, it is desirable to shape the relationships between forces and positions at both ends of the telerobotic system. Inspection of Fig. 1 reveals the following desired relationships.

1 The subscript "m" signifies the master and "s" signifies the slave. Unless otherwise noted, all variables are defined in the Laplace domain. The Laplace arguments for all functions are omitted.
between the master and slave variables that have physical significance, $y_s, y_m, f_s,$ and $f_m$.

\[ y_s = A_y y_m \]  
\[ f_s = A_f f_m \]  
\[ f_m = Z_m y_m \]  
\[ f_s = Z_s y_s \]  

$A_y$ and $A_f$ specify the amplification (or attenuation) of position and force, respectively, between the master and the slave. $Z_m$ and $Z_s$ characterize the impedances of the master and slave ports. Since the four relationships are interdependent, the entire system performance is specified when any three of these four relationships are specified. Mechanical systems are not generally responsive to commands at high frequencies. Therefore, $A_y, A_f, Z_m,$ and $Z_s$ in (1)-(4) are specified as transfer function matrices: the crossover frequencies associated with members of $A_y, A_f, Z_m,$ and $Z_s$ imply the desired frequency range in which the designers wish to operate the system. Establishing the set of performance specifications described by (1)-(4) gives designers a chance to express what they wish to have happen during a maneuver. Note that the set of specifications described by (1)-(4) does not imply any choice of control technique for a telerobotic system. We have not even said how one might achieve the above set of performance specifications. Such a set of equations only allows designers to translate their objectives into a form that is meaningful from the standpoint of control theory.

The next section introduces a practical control structure which achieves the performance specified by (1)-(4).

II. THE CONTROL ARCHITECTURE

Design of the control architecture must consider the dynamic behaviors of the master robot, the slave robot, the human operator, and the environment. These are discussed first.

A. Dynamic Behaviors of the Master Robot and the Slave Robot

It is assumed that both the master robot and the slave robot have independent closed-loop position controllers or closed-loop velocity controllers.\(^3\) Throughout this paper, this controller is called a primary stabilizing controller. The resulting closed-loop system is called a primary closed-loop system. These primary stabilizing controllers are used in both the master robot and the slave robot for the following reasons.

1) For the safety of the human operator, the master robot must remain stable when not held by a human operator. A closed-loop position controller keeps the master robot stationary when not held by an operator.

2) For the security of the object being manipulated, the slave robot must remain stable, if the communication between the slave and master is cut off accidentally. A closed-loop position controller on the slave robot keeps the slave robot stationary in these cases.

3) A closed-loop position controller creates linear dynamic behavior in the master robot and the slave robot. Here we assume that, for nonlinear robot dynamics, a nonlinear stabilizing controller has been designed to yield a nearly linear closed-loop position system for the master and slave robot. This lets us assume that the robots' closed-loop dynamics can be approximated by transfer function matrices. For brevity, the selection of this controller is not discussed here. To stabilize the master and slave robots independently, a variety of robust control methods can be used. (For several well-established robotic trajectory control techniques, refer to [23].) The closed-loop position controller also eliminates the effects of friction forces in their joints and transmission mechanisms.

4) The design of the primary stabilizing compensator lets the designers deal with the robustness of the master robot and the slave robot without getting involved in the dynamics of the human arm [24], the dynamics of the object being manipulated by the slave, or the communication time delay [2].

The derivations of the dynamic behaviors of the master robot and the slave robot are similar; as a result, only the master robot's dynamic behavior is derived here. The master robot's position $y_m$ results from two inputs: $u_m$, the desired-position command to the master's position controller, and $f_m$, the forces imposed on the master robot. $G_m$ is the primary closed-loop transfer function whose input is the desired-position command $u_m$, and whose output is the master position $y_m$. $S_m$ is the “sensitivity” transfer function whose input is the force imposed on the master $f_m$, and whose output is the master position $y_m$. Thus, (5) represents the dynamic behavior of the master robot.

\[ y_m = G_m u_m + S_m f_m. \]  

$f_m$ represents force from only the human operator, since the master robot is in contact with only the human operator. The master robot has a small response to the human force $f_m$ if the magnitude of $S_m$ is small. A small $S_m$ is achieved through the use of a high-gain closed-loop position controller as the primary controller or through the use of an actuator with a large transmission ratio [12]. One can think of $S_m$ as the disturbance rejection property of the master robot.

The dynamic behavior of the slave robot is defined by (6), which is similar to (5).

\[ y_s = G_s u_s + S_s f_s \]  

$u_s$ is the desired position command to the slave position controller, and $f_s$ is the force imposed on the slave robot endpoint by the environment. $G_s$ and $S_s$ are similar to $G_m$ and $S_m$ and represent the contributions of $u_s$ and $f_s$ on $y_s$.

B. Dynamic Behavior of the Human Arm

The dynamic behavior of the human arm's constrained, single joint movements is modeled as a functional relationship between a set of inputs and a set of outputs. As a specific example, the joint being modeled may be the elbow joint, actuated by the elbow flexor (biceps) and the elbow extensor (triceps). We avoid attributing a particular class of dynamic behaviors to constrained movements of the human arm, as

\[ y_s = G_s u_s + S_s f_s \]
it is not clear whether the arm behaves as a force or a position control system in constrained motions. Maneuvering our hands in a stream of water from one point to another target point, while struggling with the water current, is an example that shows the human arm can work as a position control in a constrained space and can continuously accept a position or velocity command from the central nervous system. Alternatively, pushing a pin into a wall is an example of a constrained movement where the human imposes a force on the pin, without being concerned with the pin position in the direction normal to the wall; this system may be viewed as one that accepts force commands from the central nervous system.

Considering the above dilemma in attributing a particular control action to constrained movements of the human arm, we use a Norton or a Thevenin equivalent concept [22] to arrive at a general substitute for the dynamic behavior of the human arm interacting with the master robot. In the same way that the choice of a Norton or Thevenin equivalent does not affect the behavior of a circuit in contact with other circuits, our choice in modeling the human arm by a Norton or a Thevenin equivalent has no effect on the arm’s interaction with other systems.

Using the “force–current” analogy between electrical and mechanical systems, a Norton equivalent is now chosen to model the human arm’s dynamic behavior as a nonideal source of force interacting with the master robot. The notion of “nonideal,” as applied here, refers to the fact that the human arm responds not only to descending commands from the central nervous system but also to position constraints imposed by interaction with the master robot. $u_h$ is that part of the contact force that is imposed by the muscles, as commanded by the central nervous system. If the human arm does not move (i.e., the arm position $y_m = 0$), the total contact force $f_m$ is the same as $u_h$. However, $f_m$ is also a function of the master robot position constraint. If the arm moves (i.e., the master robot imposes a position constraint on the human arm), the force imposed on the master robot will differ from $u_h$. The analogy can be observed from the Norton equivalent circuit shown in Fig. 2(c): the current $f_m$ is a function of not only the current source $u_h$, but also the imposed voltage $y_m$. Considering the above analogy, shown in Fig. 2, the contact force $f_h$ can be represented by

$$f_m = u_h - S_h y_m \quad (7)$$

Throughout this paper, $S_h$ is referred to as the human arm impedance and maps the master robot position constraint onto the contact force. $S_h$ is determined primarily by the physical and neural properties of the human arm. It will become clear later that $S_h$ plays an important role in the stability and performance of a telerobotic system [13]; we will arrive at some experimental values for $S_h$ in later sections. Here we give a brief description of the internal structure of the human arm impedance $S_h$ using Fig. 3. The internal structure of Fig. 2(b) is presented, where the neural feedback mechanism $G_f$ allows for regulation of the contact force $f_m$.

1) Muscle Activation Dynamics: $G_a$ produces an intended muscle force $f_i$ in response to descending neural commands $n$.

2) Muscular Contraction and Passive Tissue Dynamics: $G_p$ represents the properties of the muscles and passive tissues surrounding the joint, and alters the intended force by $G_p y_m$.

3) Neural Feedback: $G_f$ is a feedback operator regulating the force imposed by the human arm on the master robot.

The following discusses briefly the three elements listed above.

Muscle activation dynamics: $G_a$ represents muscle activation dynamics and maps the descending neural control signals into the intended muscle force $f_i$ [30]. We rely on this model, since this type of model for muscle activation dynamics has been used with some success in other work [31]. Decoupled operators for muscular activation dynamics are generally used.

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with Hill-type muscle models [29]. We are interested, however, in the overall behavior of the arm and wish to suppress the details of muscle behavior. Thus, $G_a$ is the only operator we use that explicitly describes muscle behavior. A first-order time lag has been suggested in [31] to represent $G_a$.

Muscular contraction and passive tissue dynamics: $G_p$ is an impedance that reduces the intended force, resulting in the total contact force $f_m$. $G_a$ implicitly takes into account both the internal muscle dynamics, such as the force–velocity and length–tension relationships [10], [21], [27], and the changing effective stiffness of the arm that results from varying the co-contraction of the antagonistic muscles; $G_m$ also includes the dynamic behavior of the passive tissues surrounding the joint. Equation (8) describes one possible and familiar form for $G_p$.

$$G_p = Ms^2 + Bs + K$$

where $M$ represents the mass of the human arm, and $K$ and $B$ define the visco-elastic behavior of the muscles and passive tissues.

Note that in the human arm, the descending neural control signals have two functions: 1) causing the arm to move; and 2) altering the arm impedance $G_p$. Here, we model the first of these functions by $n$ in Fig. 3. The second function, altering the arm impedance, is not explicitly shown but is accounted for in $G_p$. (See [11] for more information on neurophysiological evidence for the existence of two separate cortical systems, one for determining the commanded limb trajectory, the other for specifying the level of co-contraction.)

Neural feedback: The interaction force $f_m$ exerted by the arm on the master robot is used as a feedback signal to modulate the descending neural signals. This feedback is effective only at relatively low frequencies because of the limited bandwidth of the central nervous system. Since the human hand can apply very accurately a desired interaction force at very low frequencies, we deduce that $G_f$ is very large at low frequencies. (The experimental results, discussed in later sections, will clarify this.) On the other hand, since the human cannot apply a desired interaction force at high frequencies, $G_f$ must be very small in the high frequency range. Assuming $G_f$ is a linear transfer function, (9) is a possible choice for $G_f$.

$$\lambda \frac{C}{s + \lambda} = G_f$$

(9)

$\lambda$ is a scalar representing the bandwidth of the central nervous system, and $C$ is a feedback gain. In addition, a pure time delay operator may be introduced in order to account for neural conduction delay.

Simplifying the block diagram shown in Fig. 3 results in the transfer functions shown in Fig. 2(b).

$$f_h = G_{cns}n - S_h y_m$$

(10)

where

$$G_{cns} = \frac{G_a}{1 + G_f G_a}$$

(11)

$$S_h = \frac{G_p}{1 + G_f G_a}$$

(12)

$G_{cns}$ and $S_h$ represent, respectively, the effects of the central nervous system commands and of the master arm position $y_m$. These operators are shown in Fig. 2(b) where $u_h = G_{cns}n$. Note that Fig. 2(b) mathematically represents the closed-loop form of Fig. 3: the force imposed by the human arm on the master robot is the result of both the central nervous system commands and the master robot position constraint. Note that we do not plan to arrive at the values for $G_p$, $G_f$, and $G_a$; we will arrive at a value of $S_h$ experimentally which contains the effect of $G_p$, $G_f$, and $G_a$ implicitly.

A comparison of our modeling approach with that described in [26] clarifies our choice of modeling. In the modeling approach discussed in [26], an interaction force was modeled as the imposed disturbance, while the resultant position was modeled as the feedback variable. This modeling approach is suitable for unconstrained maneuvers. In contrast, in our modeling approach, the interaction force is modeled as the feedback variable, while the position is modeled as the imposed disturbance.

C. Dynamic Behavior of the Environment

Telerobotic systems are used for manipulating objects or imposing force on objects. Defining $E$ as a transfer function representing the dynamics of the environment and $f_{ext}$ as the equivalent of all the external forces imposed on the environment, (13) provides a general expression for the force imposed on the slave robot in the linear domain.

$$f_s = E y_s + f_{ext}$$

If the slave robot is used to push a spring and damper, as shown in Fig. 1, $E$ is a transfer function so $E(s) = (K_e + B_e s)$, and $f_{ext} = 0$, where $K_e$, $B_e$, and $s$ are the stiffness, damping, and Laplace operator, respectively.

D. The Proposed Control Architecture

The proposed control structure is shown in Fig. 4, which also represents the dynamic behaviors of the telerobotic system, the human arm, and the environment. Each dashed block represents one of the dynamic model (5), (6), (7), and (13). The information signals (the contact forces $f_m$ and $f_s$) are processed by controller $H$. The output of this controller is then fed to both drive systems, that of the master robot and that of the slave robot. Note that there is no position cross-feedback between the robots; only the contact forces are measured for feedback. This is a fundamental difference between this control structure and previous ones. (Refer to [7] and [8] for a summary of previous telerobotic control structures.)

The motion of the master robot is partially due to the transfer of human power to the robot and partially due to the command generated by the computer. Examining Fig. 4 reveals that $H_{11}$ provides additional path for $f_m$ to map to $y_m$. The physical contact between the human and the master robot produces some master robot motion as $f_m$ acts through $S_m$. In other words, the motion of the master robot is partially due to the human force on the master robot. In general, $S_m$ is much smaller than desired; thus, the human operator alone
may not have sufficient strength to move the master robot as desired. An additional route for \( f_m \) to map to \( y_m \) can be added if \( H_{11} \) is chosen to be nonzero; \( H_{11} \) can be thought of as the component that shapes the overall mapping of the force \( f_m \) to the position \( y_m \). Since the mapping \( G_m H_{11} \) acts in parallel to \( S_m \), \( H_{11} \) has the effect of increasing the apparent sensitivity of the master robot. \((G_m H_{11} + S_m)\) affects how the extender "feels" to the human operator. For instance, if \( H_{11} \) is chosen so \((G_m H_{11} + S_m)\) is approximately constant, the master robot reacts like a spring in response to \( f_m \). Similarly, if \((G_m H_{11} + S_m)\) is approximately a single or double integrator, the master robot acts like a damper or mass, respectively.

Similarly, compensator \( H_{22} \) is chosen to generate compliance in the slave robot in response to the force \( f_s \) imposed on the endpoint of the slave robot [12], [16], [17], [25]. The interaction force \( f_s \) also affects the master robot as a force reflection after passing through the compensator \( H_{12} \).

The control method presented here does not require any transfer of either position or velocity information between the master robot and the slave robot; it only requires the transfer of forces. Although this property leads to a wider communication bandwidth between the master and slave robots, the entire system may still suffer from a positional error buildup between the master robot and slave robot. This difficulty may be alleviated by periodic initialization of master and slave robots.

The communication delay between the robots and the computer becomes a major difficulty in unusual tasks where the master robot and the slave robot are very far from each other [2]. For example, if the master robot is located on the earth, while the slave robot is in orbit around the earth, then the communication delay will become a bottle neck in the system, if wide bandwidth operation is needed. Here we assume the communication bandwidth is much wider than operation bandwidth.

The goal is to find \( H \) so the chosen performance specifications given by (1)-(4) are achieved, and the stability of the system shown in Fig. 4 is guaranteed. There are many methods of achieving the performance specifications. The \( H_\infty \) control approach is chosen in this paper (described in the next section) because it captures all features described in this paper.

Fig. 4. The proposed control structure.

III. REVIEW OF THE STANDARD \( H_\infty \) CONTROL PROBLEM

Fig. 5 shows the basic block diagram used for the standard \( H_\infty \) control problem. In Fig. 5, \( P \) is the generalized plant, and \( H \) is the controller. \( P \) contains what is usually called the plant in a control system and also contains all weighting functions. The vector-valued signal \( v \) is the exogenous input, whose components are typically commands, disturbances, and sensor noises. \( e \) is the output vector to be controlled, whose components are typically tracking errors. \( u \) is the control input vector. \( x \) is the measured output vector.

In order to express the closed-loop input–output mapping from \( v \) to \( e \) as a linear fractional transformation (LFT) on \( H \), the interconnection structure \( P \) is partitioned in the following form:

\[
P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}
\]

and

\[
e = P_{11}v + P_{12}u
\]

\[
x = P_{21}v + P_{22}u.
\]

Then

\[
e = Fv
\]

\[
F = P_{11} + P_{12}H(I - P_{22}H)^{-1}P_{21}
\]

The standard \( H_\infty \) control problem is to find a stabilizing controller \( H \) which minimizes \( \|F\|_\infty \) (i.e., \( H_\infty \) norm of \( F \)), where

\[
\|G\|_\infty = \sup_\omega \sigma(|G|_\infty(j\omega)).
\]

\( \sigma(\cdot) \) denotes the maximum singular value. A detailed review of \( H_\infty \) is given in [5] and state-space results are discussed in [4] and [6]. In this algorithm \( H_\infty \) norm minimization is used to obtain a stabilizing controller \( H \) so \( \|F\|_\infty < \gamma \), where \( \gamma \) is a positive small number and may be interpreted as a measure of performance. \( \gamma \) also has the physical interpretation as being the greatest distance from the origin on a Nyquist plot of the transfer function \( F \) for all frequencies \( 0 < \omega < \infty \). Note again that the \( H_\infty \) control approach is chosen here solely due to its strength; any controller \( H \) that minimizes \( e \) within a bounded frequency range will be effective.
IV. PROBLEM FORMULATION

Depending on the application, the designer is free to choose any three of the four relationships in (1)–(4) to specify the system performance. This paper chooses $A_y$, $A_f$, and $Z_m$ [i.e., (1)–(3)] as the performance specifications for the example solution herein. (The solution obtained in this paper can be achieved similarly for all other possible combinations.) Now having fixed the performance specifications, the control problem is reduced to designing a controller that guarantees minimal deviations of the system performance from the chosen performance specifications. Equations (20)–(22) represent possible deviations in the system performance.

\[
e_1 = W_y(y_s - A_y y_m) \tag{20}
\]
\[
e_2 = W_f(f_s - A_f f_m) \tag{21}
\]
\[
e_3 = W_{zm}(y_m - Z_m^{-1} f_m) \tag{22}
\]

$W_y$, $W_f$, and $W_{zm}$ are weighting function matrices. These weighting functions are specified by designers to stress the significance of the frequency range of operation. The crossover frequencies associated with the weighting matrices must be at least equal to the desired operation bandwidth. For example, a weighting function matrix composed of transfer functions with one rad/s bandwidth implies that the designers are interested in achieving the desired performance specifications only within one rad/s.

The block diagram of Fig. 6 is derived from Fig. 4 to represent $e_1$, $e_2$, $e_3$. This block diagram is converted to the architecture of Fig. 5 by choosing

\[
e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \quad v = \begin{bmatrix} u_h \\ f_{ext} \end{bmatrix}, \quad x = \begin{bmatrix} f_m \\ f_s \end{bmatrix}, \quad u = \begin{bmatrix} u_m \\ u_s \end{bmatrix},
\]

\[
H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \tag{23}
\]

(24) and (25) below, and

\[
P_{21} = \begin{bmatrix} (I + S_h S_m)^{-1} & 0 \\ 0 & (I + E S_s)^{-1} \end{bmatrix} \tag{26}
\]

\[
P_{22} = \begin{bmatrix} -(I + S_h S_m)^{-1} S_h G_m & 0 \\ 0 & -(I + E S_s)^{-1} E G_s \end{bmatrix} \tag{27}
\]

$A_y$ and $A_f$ specify the amplification (or attenuation) of position and force, respectively, between the master robot and the slave robot. $Z_m$ characterizes the impedance of the master robot. An internally balanced realization is performed to find matrix $H$ such that $\|F\|_\infty < \gamma$ where $F$ maps $v$ to $e$ as in (17). $\gamma$ is interpreted as a measure of performance; it is designer's choice and must also satisfy the conditions described in [6], [19]. The order of the resulting controller $H$ may be large (for the experiments given in the next section, the order is 9). Since implementation of a high order controller results in a large sampling time, the order of the controller should be reduced. The model reduction technique employed here eliminates the “less significant” states of the controller and yields in a low-order realization for the controller. The full order controller should be transformed into a state-space form and the controllability and observability Gramians should be derived. Inspecting the Hankel singular values of the Gramians reveals that some of them are small compared to others. The states corresponding to those small Hankel singular values are both difficult to control and observe. In other words more energy is required to excite them and their effect on the output is also small. The states corresponding to those small Hankel singular values should be eliminated. This results in a low order controller for the system (second-order controller for the experiments of the next section).

V. EXPERIMENTS

Fig. 7 shows the experimental setup: a two-degree-of-freedom XY table was used as the master robot. A three-degree-of-freedom composite robot (shown in Fig. 8) was

\[
\begin{bmatrix}
-W_y A_y S_m (I + S_h S_m)^{-1} & W_y S_s (I + E S_s)^{-1} \\
-W_f A_f (I + S_h S_m)^{-1} S_h G_m & -W_f (I + E S_s)^{-1} E G_s \\
-W_{zm}(S_m - Z_m^{-1})(I + S_h S_m)^{-1} & W_{zm}(S_m - Z_m^{-1}) S_h G_m \\
\end{bmatrix}
\tag{24}
\]

\[
\begin{bmatrix}
-W_y A_y S_h^{-1} (I + S_h S_m)^{-1} S_h G_m & W_y S_s (I + E S_s)^{-1} S_h^{-1} G_s \\
-W_f A_f (I + S_h S_m)^{-1} S_h G_m & -W_f (I + E S_s)^{-1} E G_s \\
W_{zm}(S_h^{-1} + Z_m^{-1})(I + S_h S_m)^{-1} S_h G_m & 0 \\
\end{bmatrix}
\tag{25}
\]
The slave robot; a composite robot; high structural stiffness and low mass of the links allow for the wide bandwidth of the control system.

controller. Due to the low lead angle of the lead-screw mechanism, the XY table is not backdrivable, and therefore a human, holding a handle on the table, cannot move the table without the use of force sensors. A microcomputer is used to develop the primary stabilizing controller on the XY table. A high speed (250 kHz) AD/DA converter reads the motors’ velocity signals and the force sensors’ signals on the XY table. This converter board, which has 12 bit resolution, also sends analog command signals to the XY table servocontroller. A parallel I/O board (DID converter) between the servocontroller and the computer reads the counters’ output, which represents the XY table position.

The slave robot, incorporating a four-bar linkage, is designed so that its functional parts are balanced in all positions without the addition of counterweights [14], [15]. The motors are never loaded by gravity. As a result, smaller motors with less torque can be used to achieve higher speed, accuracy, and repeatability in fine manipulation tasks. The actuators used in this robot are neodymium (NdFeB) magnet ac brushless synchronous motors. Due to the high magnetic field strength (maximum energy products: 35 MGOe) of the rare earth NdFeB magnets, the motors have high torque-to-weight ratio. The peak torque of motors 1, 2, and 3 are 118, 78, and 58 Nm, respectively. Pancake-type resolvers are used as position and velocity sensors. Fig. 9 shows a schematic of the slave robot hardware controller:

1) Servocontroller: The servocontroller unit has an interpolator, R/D (resolver-to-digital) converter, and a computer interface; it produces three-phase, pulse-width-modulated (PWM), sinusoidal currents for the power amplifier. The unit operates in either a closed-loop velocity or current (torque) control mode. (The latter is used here.) A PWM power amplifier powers the motors with up to 47 A from a 325 V supply. The switching frequency is 4 kHz. The servocontroller has an allowable continuous current of 21 A and peak torque of 40 A.

2) Power Supply: The main dc bus power is derived from full-wave rectification of the three-phase 235 Vac incoming power and yields a dc bus voltage of 325 Vdc. The continuous power output is 18 kW.

3) Isolation Transformer: A Y to Δ isolation transformer with capacity of 20 kW is used to filter the input power. The output voltage drop from line to line is 235 Vac.
The master robot links are made of graphite-epoxy and AA7075T6 aluminum materials. A piezoelectric force sensor, which weighs 32 g, measures the environment forces ($f_r$) at the robot endpoint. The force sensor bandwidth is about 8 kHz. The force sensor allows for maximum measurement of ±500 lbf in three directions with a threshold of 0.0001 lbf. Three charge amplifiers, each with 180 kHz bandwidth, are used to convert the force sensor charge signal to voltage. This voltage is then read by an analog-to-digital converter board. Another microcomputer was employed as the primary positioning controller of the master robot. A 250 kHz, 12 bit, AD/DA converter reads the force sensor signals and sends signals to the servocontroller unit. A parallel I/O board between the servocontroller and the computer reads the R/D (resolver-to-digital) converter output.

Fig. 10 shows the environment simulator. This simulator consists of two aluminum boards. Compression-type helical springs are positioned between the stationary (labeled B) and movable metal board (labeled A) to furnish resistive force between the plates. The stationary board is mounted tight to the laboratory floor, while the movable board is maneuvered by the slave robot.

The environment simulator was set at a 10° angle with the Cartesian coordinate X-axis as shown in Fig. 11.

A. Master and Slave Robot Dynamics

Two separate microcomputers, containing high speed IO boards for communications were employed as the primary stabilizing controllers for the master robot and the slave robot. These computers communicate with each other via a parallel IO board. $H$ is implemented on the slave microcomputer.

The primary stabilizing controller for the master robot is a lead-lag controller. This controller achieves the widest bandwidth for the closed-loop position transfer function matrix $G_m$, and yet stabilizes the XY table in the presence of unmodeled dynamics. Since the XY table motion is uncoupled, $G_m$ is a 2 x 2 diagonal transfer function matrix representing the XY table dynamics in the X-direction and Y-direction (Fig. 11). The analytical form of $G_m$ was verified experimentally via a frequency response method (Fig. 12) and is given by (28).

$$G_m = \begin{pmatrix} \frac{1}{s^2 + \frac{s}{28} + 1} & 0 \\ 0 & \frac{1}{12.2^2 + \frac{s}{13.4} + 1} \end{pmatrix} \text{ cm/cm.}$$

Due to the small lead angle of the lead-screw mechanism, the XY table is not back-drivable. Therefore, the master robot cannot be moved by the force exerted on the handle by the human operator, and $S_m$ is virtually equal to zero in both the X-direction and Y-direction.

A computed torque method and a PD controller were used as the primary stabilizing controller for the slave robot. The computed torque, approximately cancels the robot nonlinear terms while the PD controller is used to decrease the error and develop robustness in modeling errors. (The details on this primary controller are omitted here. However, [1] describes this method.) This controller develops nearly an uncoupled dynamic model for the slave robot. The resulting approximate closed-loop transfer function matrix and sensitivity matrix (as seen in (29) and (30) on the bottom of the following page) are for the master robot.
The human arm model derived here does not represent the human arm sensitivity $S_h$ for all configurations, but is only an approximate and experimentally verified dynamic model of the operator’s arm in the neighborhood of the operating configuration shown in Fig. 8. In the identifying process, the operator was seated next to the master robot while grasping the handle with her right hand as shown in Fig. 8. The master robot was commanded to oscillate in a sinusoidal fashion along the X and Y axes, respectively. At each oscillation frequency, the operator attempted to move her arm to follow the master robot so that no contact force between her hand and the master robot was generated [i.e., she decided not to impose any force on the master robot ($u_h = 0$)]. Since the human arm cannot keep up with any high frequency movement of the master when trying to maintain zero contact force, a large contact force and consequently a large $S_h$ are expected at high frequencies. Since this force is equal to the product of the master acceleration and the human arm inertia (Newton’s Second Law), a second-order transfer function is expected at high frequencies. These data agree with previous results [18], [26], in which passive wrist motion was shown to be second-order and dominated by the moment of inertia. At low frequencies, however, the human can follow the large motions of the master robot quite comfortably, but it is expected that some finite contact force is present. Therefore, the human arm sensitivity $S_h$ approaches a finite value at low frequencies. In comparing the data, we have observed that the human arm impedance at low co-contraction levels is significantly smaller than it is at high co-contraction levels in the low frequency range. These results are hardly surprising, given the evidence discussed above that both stiffness and viscosity are roughly proportional to mean muscle tension and that co-contraction has an especially strong influence on both parameters at low frequencies [28]. At high frequencies, we assume that the forearm is moved without the benefit of the active feedback loop $G_f$. Based upon the experimental data, the best estimates for the author’s arm sensitivities along the X and Y axes are as in (31) below.

Crossover frequencies in experimental $S_h$ of 3–6 rad/s were observed. The crossover frequency is the maximum frequency at which the subject was able to accurately control the constrained movement, for a given co-contraction level. The crossover frequency creates the distinction between the low and high frequency regions of operation, and is itself defined by the time delays associated with finite neural conduction velocities.

In the experiments described here, the human elbow joint was made to operate solely within the mid-portion of its full range of motion. Thus, we avoided significant nonlinearities associated with joint torques caused by passive tissues around the joint. It has been shown that passive elastic torques in the human wrist are small and relatively constant within 40 degrees in either direction from the middle of the wrist’s range.

\[
G_s = \begin{bmatrix}
1690.9 & 0 \\
0 & 1503.6
\end{bmatrix} \text{cm/cm}
\]

\[
S_s = \begin{bmatrix}
89.19 & 0 \\
0 & 12.08
\end{bmatrix} \text{cm/N.}
\]

\[
S_h = \begin{bmatrix}
0.1 \left( \frac{s^2}{2.52} + \frac{s}{2.19} + 1 \right) & 0 \\
0 & 0.125 \left( \frac{s^2}{2.752} + \frac{s}{1.83} + 1 \right)
\end{bmatrix} \text{N/cm.}
\]
of motion, and rise rapidly at either extreme of the full range of motion [18]. Similar results were observed in the human elbow joint [9], where the passive viscous torque was found to vary by a factor of approximately 5, with an obvious minimum at a point within the midrange of the joint.

C. Environment Dynamics

Fig. 10 shows the environment simulator. This simulator consists of two metal boards. Compression-type helical springs are positioned between the stationary and movable metal boards to furnish a resistive force between the plates. The stationary board is mounted securely on the floor. The dynamic model of the movable plate is expressed by (32).

\[ E = 21.582 \text{ N/cm}. \] (32)

\( E \) is defined by (13).

The environment simulator was set at a 100° angle with the Cartesian coordinate X-axis as shown in Fig. 11. This angle was introduced purposely into the experiment to develop kinematic coupling. Although both of the master and slave robots have uncoupled dynamic behaviors, the 100° angle leads to a nondiagonal dynamics and consequently a nondiagonal controller \( H \) matrix.

The chosen performance specifications for \( A_f \) and \( Z_m \) are given by the following equations:

\[ A_f = \begin{bmatrix} A_{f_x} & 0 \\ 0 & A_{f_y} \end{bmatrix}, \] (33)

\[ Z_m = \begin{bmatrix} Z_{m_x} & 0 \\ 0 & Z_{m_y} \end{bmatrix} \] (34)

where

\[ Z_{m_x} = Z_{m_y} = 4s + 0.04 \text{ N/cm} \] (35)

and

\[ A_{f_x} = -0.5 \text{ and } A_{f_y} = -1.5. \] (36)

The above choice of performance specification leads to force amplification of 1.5 in the \( Y \) direction and force attenuation of 0.5 in the \( X \) direction. In other words, in the \( Y \) direction, the force felt by the human operator is larger than the slave force by a factor of 1.5. At the same time, in the \( X \) direction, the human feels only 50% of the slave forces. Note that the chosen form for \( Z_m \) in (35) contains both viscous and stiffness components. The viscous element (i.e., 4 s) has been shown experimentally to be a comfortable impedance for the operator. The small stiffness component (i.e., 0.04) returns the master robot back to its zero condition very slowly when not held by the human. The elements of all weighting functions were chosen as \((0.01/s + 1)\).

An internally balanced realization is performed to find matrix \( H \) that minimizes \( \|F\|_{\infty} \) where \( F \) maps \( v \) to \( e \) as in (17). The order of the resulting controller \( H \) was 9. Inspection of the Hankel singular values of the Gramian matrix of the full order \( H \) reveals that some of them are small compared to others. The states corresponding to those small Hankel singular values were eliminated. This results in (37) (see below) for controller \( H \).

Fig. 14 shows the slave force and the master force in the \( X \)-direction. Fig. 15 shows the slave force versus the master force where the slope of the fitted curve confirms the achievement of the desired force attenuation in the \( X \)-direction. Note the coupling between the forces in the \( X \) and \( Y \) directions. This coupling is due to the residual nonlinearities in the slave robot; although the computed torque method in conjunction with the PD controller usually leads to linear behavior, the high speed of the maneuvers lead to some degree of coupling. Figs. 16 and 17 are similar to Figs. 14 and 15 and show the force amplification in the \( Y \)-direction.

\[
H = \begin{bmatrix}
0.25s + 3.75 & 0.5s + 7.5 \\
0.125s + 1.87 & -2.29s + 3.72 \\
\end{bmatrix}
\begin{bmatrix}
s^2 + 15s + 0.15 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
0.25s + 2.5 & 0.167s + 1.66 \\
0.37s + 3.74 & -0.05s + 2.5 \\
\end{bmatrix}
\begin{bmatrix}
s^2 + 10s + 0.1 \\
0 \\
\end{bmatrix}
\] (37)
slope = -0.54

Fig. 15. The slope of the fitted linear curve -0.54 confirms the force amplification of $A_{fx} = -0.5$ in the $x$ direction.

Fig. 16. Plots of master and slave forces in the $Y$-direction.

The entire work presented here is framed by linear system theory. The following example shows a scenario where the control method described in this paper may fail and one requires a nonlinear control approach. In movements along an arbitrary trajectory, the goal may be a dynamic behavior for the telerobotic system; the human who is maneuvering a rigid body feels the forces as being those of maneuvering a light single-point mass. This dynamic behavior masks the cross-coupled forces associated with maneuvering a rigid body; the human feels only the forces associated with the acceleration of a single-point mass. This behavior is desirable, because cross-coupled forces contribute to the difficulty of maneuvering a rigid body. For the example above, the relationship between the forces on the master robot and the forces on the slave robot cannot be given explicitly by (1)–(4). We hope that future research work on telerobotics will be focused on control techniques that will allow any arbitrary nonlinear design specifications.

VI. SUMMARY AND CONCLUSION

This paper presents a design framework for telerobotic systems that can achieve desired dynamic relationships between the master robot and the slave robot. A minimum number of linear transfer functions are defined to frame the design specifications. The human arm, in constrained movements, is in continuous contact with the master robot. The defining characteristic of the human arm in such movements is associated with the ability of the human to impose desired forces. We have given a theoretical and experimental analysis for constrained movements of the human arm. Using the models for the human arm, the robots and the load being maneuvered, $H_{nc}$ control theory and model reduction techniques are used to guarantee that the system behavior is governed by the proposed specified functions. The system may suffer from a positional error buildup between the master robot and slave robot, since the control method presented here transfers neither position nor velocity information between the master robot and the slave robot. Several experiments were carried out to verify the theoretical derivations.

NOMENCLATURE

$A_f$ $m \times m$ transfer function matrix; desired force amplification.

$A_v$ $m \times m$ transfer function matrix; desired position amplification.

$B$ Viscosity in $G_p$.

$B_e$ Damping of the environment.

$C$ dc feedback gain.

$E$ $m \times m$ transfer function matrix; environment impedance.

$3m \times 1$ vector.
\( f_{\text{ext}} \) \( m \times 1 \) vector; any force imposed on the environment other than by slave robot.

\( f_i \) The intended force applied by the human arm, as commanded by the central nervous system.

\( f_m \) \( m \times 1 \) vector; force imposed on the master robot by a human.

\( f_s \) \( m \times 1 \) vector; force imposed on the slave robot by an environment.

\( G_a \) Transfer function; describing muscle activation dynamics.

\( G_{\text{cns}} \) Transfer function; relating neural input to the human arm configuration.

\( G_f \) Transfer function; feedback operator whose output affects \( n_s \), the neural control signals.

\( G_m \) \( m \times m \) transfer function matrix; primary closed-loop position control of the master robot.

\( G_p \) Transfer function; representing the inertial and visco-elastic properties of the muscles and passive tissues surrounding the joint.

\( G_s \) \( m \times m \) transfer function matrix; primary closed-loop position control of the slave robot.

\( H \) \( 2m \times 2m \) transfer function matrix; controller.

\( K \) Stiffness in \( G_p \).

\( K_e \) Stiffness of the environment.

\( M \) Inertia in \( G_p \).

\( m \) Number of degrees of freedom in the master robot and in the slave robot.

\( n \) Neural input to the muscles.

\( P \) \( 5m \times 5m \) transfer function matrix, plant.

\( S_h \) \( m \times m \) transfer function matrix; sensitivity of the human arm to the imposed motion (impedance).

\( S_m \) \( m \times m \) transfer function matrix; sensitivity of the master robot with a closed-loop position controller to the imposed force.

\( S_s \) \( m \times m \) transfer function matrix; sensitivity of the slave robot with a closed-loop position controller to the imposed force.

\( u \) \( m \times 1 \) vector, \( [u_m \quad u_s]' \).

\( u_h \) \( m \times 1 \) vector; the human muscle force which initiates a maneuver.

\( u_m \) \( m \times 1 \) vector; desired position of the master robot.

\( u_s \) \( m \times 1 \) vector; desired position of the slave robot.

\( v \) \( 2m \times 1 \) vector, \( [v_h \quad f_{\text{ext}}]' \).

\( W_f \) \( m \times m \) transfer function matrix; weighting function.

\( W_y \) \( m \times m \) transfer function matrix; weighting function.

\( W_{ym} \) \( m \times m \) transfer function matrix; weighting function.

\( x \) \( 2m \times 1 \) vector, \( [f_m \quad f_s]' \).

\( y_m \) \( m \times 1 \) vector; position of the master robot.

\( y_s \) \( m \times 1 \) vector; position of the slave robot.

\( Z_m \) \( m \times m \) transfer function matrix; desired impedance of the master robot.

\( Z_s \) \( m \times m \) transfer function matrix; desired impedance of the slave robot.

\( \alpha \) Scalar, smaller than unity.

\( \beta \) Scalar, greater than unity.

\( \gamma \) Low-pass filter transfer function.

\( \lambda \) Scalar.

REFERENCES


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