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Dynamic Behavior of Pneumatic Systems for Lower Extremity Extenders

J. M. Tressler, T. Clement, H. Kazerooni, M. Lim
University of California at Berkeley
Berkeley, CA 94720
kazerooni@me.berkeley.edu

Abstract— **This article models a pneumatic system consisting of double-acting or single acting cylinder and a servovalve with the goal of providing an insight into pneumatic design and control requirements for Berkeley Exoskeleton (http://www.me.berkeley.edu/hel/). The modeling approach uses the thermodynamic principles of energy and mass conservation. We demonstrate that pneumatic systems suffer from two effects: unwanted dynamics due to gas compressibility and discontinuous nonlinearties in the servovalves due to choked flow. To obtain a wide control bandwidth, one needs to model these effects thoroughly.**

Keywords—**Pneumatics Modeling, Control, Exoskeletons**

I. Introduction

A lower limb exoskeleton is a wearable device that assists a human to carry a load while walking. The machine is anthropomorphic and is attached at various points along the operator's legs and torso such that the geometry of the human and the machine approximately match one another. The actuation system presented in this paper is to power a lower limb exoskeleton. Most of the problems associated with the pneumatic actuation stem from the gas compressibility and its effects on both actuator and servovalve. Similar to hydraulic systems, we will derive two sets of equations: one set of equations for the actuator and one set of equations for the servovalve. These equations in conjunction with the equations of motion for the load (e.g. Newton's law) will characterize the dynamic behavior of pneumatic systems. We have verified the integrity of the equations through extensive experiments.

II. Derivation of the Dynamics of the Actuator

The purpose of this section is to derive a set of equations for the dynamic behavior of the pneumatic cylinder. The system consists of a double-acting linear pneumatic cylinder fed by a 4-way servo valve as shown in figure 1. Considering a control volume that encompasses both chambers of the cylinder, the first law of thermodynamics, at the instance that chamber 1 is the intake chamber can be written as equation 1:

$$\dot{Q}+\dot{m}_1\left(h_{enter}+\frac{v^2{}_{enter}}{2}\right)=\dot{m}_2\left(h_{exit}+\frac{v^2{}_{exit}}{2}\right)+\frac{\partial E}{\partial t}+\dot{W} \qquad (1)$$

where:

- $\dot{Q}$ is the heat rate to the control volume.

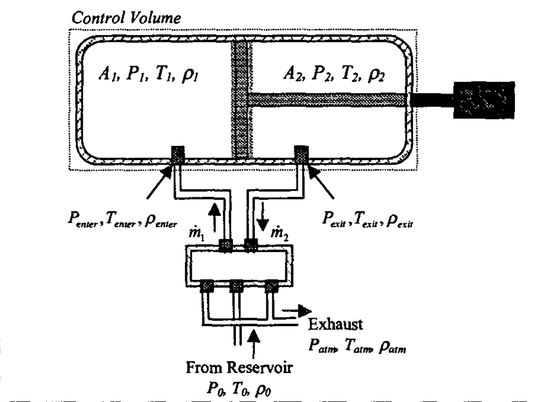

FIG. 1. Pneumatic Actuation System

- $\dot{W}$ is the work rate (power) delivered by the control volume to the piston assembly.
- $\frac{\partial E}{\partial t}$ is the rate of change of the total energy of the control volume (both chambers).
- $\dot{m}_1$ is the mass flow rate entering the control volume.
- $\dot{m}_2$ is the mass flow rate exiting the control volume
- h_{enter} is the enthalpy of the gas entering chamber 1.
- v_{enter} is the velocity of the gas entering chamber 1.
- h_{exit} is the enthalpy of the gas exiting chamber 2.
- v_{exit} is the velocity of the gas exiting chamber 2.

A. Evaluation of $\frac{\partial E}{\partial t}$

The rate of change in kinetic and potential energies of the control volume are assumed small in comparison to the rate of change of the corresponding internal energy and are therefore omitted. Therefore the rate of change of the total energy of the control volume is:

$$\frac{\partial E}{\partial t}=\frac{\partial(U_1)}{\partial t}+\frac{\partial(U_2)}{\partial t} \qquad (2)$$

where U_1 and U_2 are the internal energies of chamber 1 and chamber 2, respectively, and are defined by equation 3 and equation 4 assuming ideal gas is used for the system.

$$U_1 = C_V \rho_1 V_1 T_1 \tag{3}$$

$$U_2 = C_V \rho_2 V_2 T_2 \tag{4}$$

where V_1, T_1, ρ_1, V_2, T_2 and ρ_2 are volume, temperature and density associated with chambers 1 and 2 respectively. Alternatively equations 3 and 4 can be written as:

$$U_1 = \left(\frac{C_V}{R}\right) P_1 V_1 \tag{5}$$

$$U_2 = \left(\frac{C_V}{R}\right) P_2 V_2 \tag{6}$$

where P_1 and P_2 are the pressures in chambers 1 and 2. C_V and R are the *specific heat at constant volume* and the *gas constant*, respectively. Substituting for U_1 and U_2 from equations 5 and 6 into equation 2 results in equation 7 for the rate of change of energy of the control volume.

$$\frac{\partial E}{\partial t} = \left(\frac{C_V}{R}\right)\left(\dot{P}_1 V_1 + P_1 \dot{V}_1\right) + \left(\frac{C_V}{R}\right)\left(\dot{P}_2 V_2 + P_2 \dot{V}_2\right) \tag{7}$$

B. Evaluation of $\dot{W}$

The work rate done on the piston assembly by the gas in the pneumatic actuator is:

$$\dot{W} = P_1 \dot{V}_1 + P_2 \dot{V}_2 \tag{8}$$

where $\dot{V}_1$ and $\dot{V}_2$ are the rates of change of volume of chambers 1 and 2.

C. Evaluation of $\dot{m}_1\left(h_{enter} + \frac{v_{enter}^2}{2}\right)$ and $\dot{m}_2\left(h_{exit} + \frac{v_{exit}^2}{2}\right)$

The gas entering to the actuator comes from a reservoir (usually an accumulator). Since, the gas in the reservoir has zero velocity, its enthalpy is represented by the stagnation enthalpy h_0. Equation 9 describes the relationship between the stagnation enthalpy h_0 and the enthalpy of the gas entering chamber 1.

$$h_{enter} + \frac{v_{enter}^2}{2} = h_0 = C_P T_0 \tag{9}$$

where T_0 is the temperature of the gas in the accumulator (C_P *is specific heat at constant pressure* and is related to the aforementioned gas constants by $C_P = C_V + R$.) Similarly, the gas in chamber 2 has a very small velocity in comparison with the gas that is exiting through the servovalve with the velocity of v_{exit}. With this assumption, equation 10 relates the enthalpy of chamber 2, h_2, to the enthalpy of the gas exiting the valve.

$$h_{exit} + \frac{v_{exit}^2}{2} = h_2 = C_P T_2 \tag{10}$$

where T_2 is the temperature of the gas in chamber 2. Note the duality between equations 9 and 10 in that the intake and exhaust enthalpies are based solely on their upstream gas temperatures.

D. Reevaluation of Equation 1

Substituting for $\partial E/\partial t$ from equation 7 and for the entering and existing energies from equations 9 and 10 into 1 results in equation 11:

$$\begin{aligned}\dot{Q} + \dot{m}_1 (C_P T_0) = \dot{m}_2 (C_P T_2) \\ + \left(\frac{C_V}{R}\right)\left(\dot{P}_1 V_1 + P_1 \dot{V}_1\right) + \left(\frac{C_V}{R}\right)\left(\dot{P}_2 V_2 + P_2 \dot{V}_2\right) + \dot{W}\end{aligned} \tag{11}$$

Further substitution for $\dot{W}$ from equation 8 into equation 11 and simplification of terms results in equation 12.

$$\begin{aligned}\dot{Q} + \dot{m}_1 (C_P T_0) = \dot{m}_2 (C_P T_2) + \left(\frac{C_V}{R}\right)\left(\dot{P}_1 V_1 + \dot{P}_2 V_2\right) \\ + \left(1 + \frac{C_V}{R}\right)\left(P_1 \dot{V}_1 + P_2 \dot{V}_2\right)\end{aligned} \tag{12}$$

Assuming very little heat exchange with its surrounding, equation 13 represents the first law for the actuator.

$$\dot{m}_1 (R T_0) = \dot{m}_2 (R T_2) + \left(\frac{1}{k}\right)\left(\dot{P}_1 V_1 + \dot{P}_2 V_2\right) + \left(P_1 \dot{V}_1 + P_2 \dot{V}_2\right) \tag{13}$$

where $k = C_P / C_V$.

Taking the origin ($X = 0$) to be the far left end of the actuator and the actuator stroke length as L, the chamber volumes are:

$$\begin{cases} V_1 = A_1 X \\ V_2 = A_2 (L - X) \end{cases} \tag{14}$$

The derivatives of the chamber volumes are:

$$\begin{cases} \dot{V}_1 = A_1 \dot{X} \\ \dot{V}_2 = -A_2 \dot{X} \end{cases} \tag{15}$$

Substituting from 14 and 15 in equation 13 results in equation 16.

$$\dot{m}_1(RT_0)-\dot{m}_2(RT_2)=\left(\frac{1}{k}\right)(\dot{P}_1A_1-\dot{P}_2A_2)X + \left(\frac{1}{k}\right)\dot{P}_2A_2L+(P_1A_2-P_2A_2)\dot{X} \quad (16)$$

Equation 16 states how X varies as one modulates the mass flow rates $\dot{m}_1$ and $\dot{m}_2$ as a function of the valve opening. Derivation of $\dot{m}_1$ and $\dot{m}_2$ requires the derivation of the dynamics of the pneumatic servo vales.

III. Derivation of Dynamics of the Servo Valves

The objective of this section is to calculate the mass flow rate $\dot{m}_1$ and $\dot{m}_2$ as a function of the valve opening A_t since A_t is the control variable. A converging nozzle fed from a large reservoir is considered as a good model for the valve. This converging passage discharges into chamber 1 of the cylinder where the pressure is P_1 (Figure 2). It is assumed that the gas flow is adiabatic everywhere in the valve. It is also assumed that the flow is isentropic everywhere except across normal shock waves.

The possible flow patterns in the valve can now be investigated depending on the values of the cylinder pressure, P_1 and the supply pressure P_S.

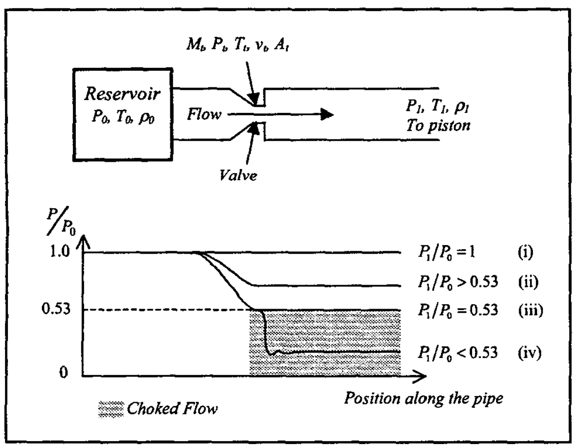

Fig. 2. A converging nozzle fed from a large reservoir is considered as a good model for the valve. Other types (e.g. converging-diverging nozzels) were also investigated, however the above converging nozzle, through experimental observation, was proven to be the most accurate model.

Case 1: No-flow condition; $\frac{P_1}{P_0}=1$

In this case, the cylinder pressure, P_1, and the supply pressure, P_0, are equal. No flow takes place in the valve from supply pressure to the cylinder. This is a case where the load on the piston is so large that the piston will not move when the valve is fully open.

Case 2: Sub-critical flow regime; $0.53<\frac{P_1}{P_0}<1$

If the valve is opened slightly, then there will be a flow with a constantly decreasing pressure through the nozzle. Since the flow is subsonic at the exit plane, the throat pressure P_t must be the same as the cylinder pressure P_1. It has been shown experimentally that the pressure in the pipe from the valve down to the cylinder is equal to the cylinder pressure. So, almost from the end of the valve down to the cylinder chamber, the pressure is uniform and equal to P_1.

Case 3: Critical flow regime $\frac{P_1}{P_0}=0.53$

As the difference between the supply pressure and the cylinder pressure increases, the stream velocity at the throat increases, until the point where the flow reaches its critical regime. At this point, the velocity of the gas in the throat is equal to the speed of sound calculated at the throat, and would never get larger even if the pressure difference increases.

Case 4: Supercritical flow regime $\frac{P_1}{P_0}<0.53$

Further reducing in the cylinder pressure will not affect the flowing state at the throat because the flow is choked in the valve. In this regime, the pressure of the jet leaving the nozzle is greater than the cylinder pressure P_1. The sudden reduction in the pressure causes the jet to expand in an explosive fashion. The pressure at throat P_t stays constant equal to $0.53P_0$. This situation is quite common and occurs when there is little load on the piston and when P_1 is much smaller than P_0.

Our experiments also showed that when the gas flow is under-choked, the pressure at the throat and the pressure in the cylinder are the same. When the gas flow gets choked, the pressure at the throat stays constant and equal to $0.53P_0$, whereas the cylinder pressure can decrease more. The derivation of $\dot{m}_1$ and $\dot{m}_2$ as function of the gas properties at the throat is straightforward. But the derivation of $\dot{m}_1$ and $\dot{m}_2$ as a function of the cylinder pressure needs to be developed for two cases: choked or under-choked gas flow in the servovalve. The values for the pressure, density, and temperature of the gas flowing through the throat of the valve can be calculated from equations 17, 18, and 19, regardless of the flow condition in the valve [4].

$$\frac{P_t}{P_0}=\left(1+\frac{k-1}{2}M_t^2\right)^{\frac{k}{1-k}} \quad (17)$$

$$\frac{\rho_t}{\rho_0}=\left(1+\frac{k-1}{2}M_t^2\right)^{\frac{1}{1-k}} \tag{18}$$

$$\frac{T_t}{T_0}=\left(1+\frac{k-1}{2}M_t^2\right)^{-1} \tag{19}$$

The definition of the mass flow rate is:

$$\dot{m}_1=\rho_t A_t v_t \tag{20}$$

where ρ_t and v_t are the density and velocity of the gas at the throat. The velocity of the gas flow at throat can be calculated from the definition of the Mach number M_t as follows:

$$v_t=M_t\cdot\sqrt{kRT_t} \tag{21}$$

Substituting equations 17, 18, 19 and 21 into equation 20 gives the following expression for flow rate in terms of the valve opening, the Mach number of the flow at the throat, and the reservoir properties:

$$\dot{m}_1=M_t\left(1+\frac{k-1}{2}M_t^2\right)^{\frac{k+1}{2(1-k)}}\cdot\rho_0\cdot\sqrt{kRT_0}\cdot A_t \tag{22}$$

or:

$$\dot{m}_1=M_t\left(1+\frac{k-1}{2}M_t^2\right)^{\frac{k+1}{2(1-k)}}\cdot\sqrt{\frac{k}{RT_0}}\cdot P_0\cdot A_t \tag{23}$$

The value of $\dot{m}_1$ depends on the exact knowledge of the Mach number at throat M_t, and the valve opening A_t. Inverting equation 17 gives the expression of the Mach number as a function of the pressure at the throat P_t:

$$M_t=\sqrt{\frac{2}{k-1}}\cdot\left(\left(\frac{P_t}{P_o}\right)^{\frac{1-k}{k}}-1\right)^{\frac{1}{2}} \tag{24}$$

Substituting for M_t from equation 24 into equation 23 results in equation 25 for mass flow rate as a function of the gas properties in the reservoir, the pressure at throat, and the throat opening:

$$\dot{m}_1=\sqrt{\frac{2}{k-1}}\left(\frac{P_t}{P_0}\right)^{\frac{k+1}{2k}}\cdot\left(\left(\frac{P_t}{P_0}\right)^{\frac{1-k}{k}}-1\right)^{\frac{1}{2}}\cdot\sqrt{\frac{k}{RT_0}}\cdot P_0\cdot A_t \tag{25}$$

Expression 25 is valid for gas going in the cylinder, whether the flow is choked or under-choked. We now need to eliminate the only remaining unknown P_t. In order to do this, let's define γ as:

$$\gamma=\sqrt{\frac{2}{k-1}}\cdot\left(\frac{P_t}{P_0}\right)^{\frac{k+1}{2k}}\cdot\left(\left(\frac{P_t}{P_0}\right)^{\frac{1-k}{k}}-1\right)^{\frac{1}{2}} \tag{26}$$

Therefore equation 25 for mass flow rate is written as:

$$\dot{m}=\gamma\cdot\sqrt{\frac{k}{RT_0}}\cdot P_0\cdot A_t \tag{27}$$

Now we have to consider two cases:

Case 1: Under-choked gas flow

When the flow is under-choked, the pressure at the throat is equal to the cylinder pressure. Therefore γ_1 is given by equation 26:

$$\gamma_1=\sqrt{\frac{2}{k-1}}\cdot\left(\frac{P_1}{P_0}\right)^{\frac{k+1}{2k}}\cdot\left(\left(\frac{P_1}{P_0}\right)^{\frac{1-k}{k}}-1\right)^{\frac{1}{2}} \tag{28}$$

Case 2: Choked gas flow

When the throat gets choked, the pressure at the throat stays constant, equal to $0.53P_0$. In this case, γ_1 stays constant.

$$\gamma_1=0.58 \tag{29}$$

Therefore, the expression of the mass flow rate for gas going into the cylinder depends on two cases, whether the flow is choked or under-choked. Expression 30 summarizes the above results:

$$\dot{m}_1=\gamma_1\cdot\sqrt{\frac{k}{RT_0}}\cdot P_0\cdot A_t \tag{30}$$

where, if $P_1>0.53P_0$ (under-choked), then

$$\gamma_1=\sqrt{\frac{2}{k-1}}\cdot\left(\frac{P_1}{P_0}\right)^{\frac{k+1}{2k}}\cdot\left(\left(\frac{P_1}{P_o}\right)^{\frac{1-k}{k}}-1\right)^{\frac{1}{2}}$$

and, if $P_1\leq 0.53P_0$ (choked), then

$$\gamma_1=0.58$$

This factor γ_I can be illustrated as part of the 'gain' of the throat opening in the mass flow rate expression.

The outgoing mass flow rate can be derived similarly, but the second chamber is now taken as a "reservoir".

$$\dot{m}_2 = \gamma_2 \cdot \sqrt{\frac{k}{RT_2}} \cdot P_2 \cdot A_t \tag{31}$$

7where, if $P_{atm} > 0.53P_2$ (under-choked), then

$$\gamma_2 = \sqrt{\frac{2}{k-1}} \cdot \left(\frac{P_{atm}}{P_2}\right)^{\frac{k+1}{2k}} \cdot \left(\left(\frac{P_{atm}}{P_2}\right)^{\frac{1-k}{k}} - 1\right)^{\frac{1}{2}}$$

and, if $P_{atm} \leq 0.53P_2$ (choked), then

$$\gamma_2 = 0.58$$

To stay consistent with the conventions we have made before, we will consider A_t as an algebraic area, which will be positive for gas going in and out of the actuator.

IV. Complete Dynamic Equations of the Actuator and Valve

The goal of this section is to summarize the necessary equations for control of a pneumatic system. The equation for actuator force is expressed as:

Newton's Law:

$$J\ddot{X} = P_1A_1 - P_2A_2 \tag{32}$$

where J is the load mass.

Actuator's Equation

$$\dot{m}_1(RT_0) - \dot{m}_2(RT_2) = \left(\frac{1}{k}\right)(\dot{P}_1A_1 - \dot{P}_2A_2)X + \left(\frac{1}{k}\right)\dot{P}_2A_2L + (P_1A_2 - P_2A_2)\dot{X} \tag{16}$$

Valve Equation:

Equations 30 and 31 or one can use equation 33:

$$\dot{m}_1(RT_0) - \dot{m}_2(RT_2) = \left(\gamma_1\sqrt{kRT_0}P_0 - \gamma_2\sqrt{kRT_2}P_2\right)A_t \tag{33}$$

where γ_1 and γ_2 can be calculated from equations 26 and 28 depending on values of $\frac{P_1}{P_0}$ and $\frac{P_{atm}}{P_2}$.

For example, for force control, assuming $F = P_1A_1 - P_2A_2$, equations 16 and 33 result in the following equation for F as a function of valve opening, A_t:

$$\left(\gamma_1\sqrt{kRT_0}P_0 - \gamma_2\sqrt{kRT_2}P_2\right)A_t = \left(\frac{1}{k}\right)\dot{F}X + \left(\frac{1}{k}\right)\dot{P}_2A_2L + F\dot{X} \tag{34}$$

This paper is focusing on the proper dynamic equations for the valves and actuators only; there are a great number of control algorithms that can be found to control the above system.

V. Design Equations

This section describes some interesting results that can be used for design and component selection. Usually in design of pneumatic systems, practitioners face the following question: For a given supply pressure, how much gas flow does one need to deliver a particular $\bar{\dot{W}}$ average power?

Equation 13 at steady-state where $\dot{m}_1 = \dot{m}_2$ and $\dot{P}_1 = \dot{P}_2 = 0$ can be written as:

$$\dot{m}_1R(T_0 - T_2) = \bar{\dot{W}} \tag{35}$$

The mass flow rate from compressor is:

$$\dot{m}_1 = \rho_0Q_0 \tag{36}$$

Substituting for $\dot{m}_1$ from equation 36 into equation 35 results in the following equation for volumetric flow rate Q_0:

$$\rho_0Q_0R(T_0 - T_2) = \bar{\dot{W}} \tag{37}$$

or

$$Q_0P_0 = Q_0P_2 + \bar{\dot{W}} \tag{38}$$

The term Q_0P_0 is called the "hydraulic power" even though we are dealing with gas. Hydraulic power means the energy associated with the gas without considering its sensible energy. Equation 38 indicates that from the hydraulic power coming to the actuator (Q_0P_0), the amount of Q_0P_2 is wasted. In other words, an actuator receives energy of Q_0P_0, but since it releases gas at pressure of P_2 it therefore wastes Q_0P_2.

Example: To extract 1 Hp work from a supply pressure of 250 psig, how much gas flow is needed? One needs to have a good guess for P_2. Large values of P_2 indicate large losses. A good dynamic simulation of the system with a proper controller usually leads for a good estimate for P_2. Assuming P_2 is about half of P_0, the flow rate needed to produce 1 Hp is:

$$Q_0 = \frac{\dot{W}}{P_0 - P_2} = \frac{(1\,\text{Hp})\left(550\,\frac{\text{lb·ft}}{\text{sec·Hp}} \cdot 12\,\frac{\text{in}}{\text{ft}}\right)}{250\,\text{psig} - 125\,\text{psig}} = 52.8\,\frac{\text{in}^3}{\text{sec}} \tag{39}$$

Evaluating the hydraulic power of the above example shows that 1 Hp work is lost due to backpressure in the cylinder. Minimizing these losses increases efficiency of the system with regards to the hydraulic power. Note that the above method only gives a size for the average flow rate. The peak flow rate is assigned by accumulator design.

VII. Single-Acting Actuator

The same methodology used to derive equation 34 for a double-acting actuator is also used for single-acting actuators. The main difference is that chamber 2 is always at atmospheric pressure. Considering the single acting pneumatic actuator of figure 3 and the nomenclature shown below, the following set of equations can be used to describe the cylinder behavior.

- P_P: Cylinder Pressure
- P_O, T_O: Supply Pressure and Temperature
- A_P: Piston Area
- A_t: Servovalve opening
- X_P: Piston position
- P_{atm}: Atmospheric Pressure

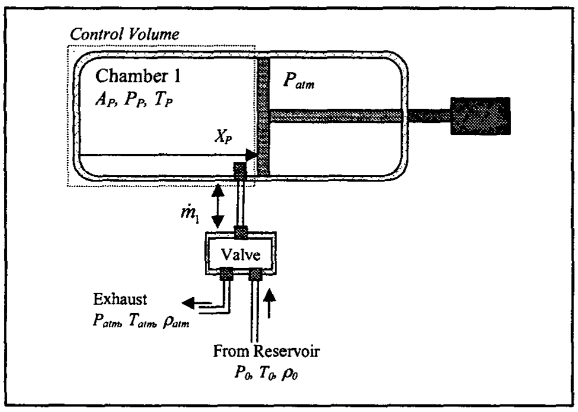

Fig. 3. Single Chamber Pneumatic Actuation System

Gas going into cylinder

$$\dot{P}_P X_P + k P_P \dot{X}_P = \dot{m}\frac{kR}{A_P}T_0 \tag{40}$$

$$\dot{m} = \gamma \cdot \sqrt{\frac{k}{RT_0}} \cdot P_0 \cdot A_t \tag{41}$$

If $P_p > 0.53P_0$ (under-choked) then

$$\gamma = \sqrt{\frac{2}{k-1} \cdot \left(\frac{P_p}{P_0}\right)^{\frac{k+1}{2k}} \cdot \left(\left(\frac{P_p}{P_o}\right)^{\frac{1-k}{k}} - 1\right)^{\frac{1}{2}}}$$

If $P_p \le 0.53P_0$ (choked) then $\gamma = 0.58$

Gas leaving the cylinder

$$\dot{P}_P X_P + k P_p \dot{X}_P = \dot{m}\frac{kR}{A_P}T_p \tag{42}$$

$$\dot{m} = \gamma \cdot \sqrt{\frac{k}{RT_p}} \cdot P_p \cdot A_t \tag{43}$$

If $P_{atm} > 0.53P_p$ (under-choked) then

$$\gamma = \sqrt{\frac{2}{k-1} \cdot \left(\frac{P_{atm}}{P_p}\right)^{\frac{k+1}{2k}} \cdot \left(\left(\frac{P_{atm}}{P_p}\right)^{\frac{1-k}{k}} - 1\right)^{\frac{1}{2}}}$$

If $P_{atm} \le 0.53P_p$ (choked) then $\gamma = 0.58$

And Newton's law:

$$J\,\ddot{X}_p = A_p(P_p - P_{atm}) \tag{44}$$

where J is the load mass.

VIII. Conclusions

This article has presented the dynamic models for a pneumatic cylinder and servovalve and incorporated them together to develop criteria for both design and control of an exoskeleton system being designed at Berkeley. The equations have the most important characteristics needed to model pneumatic systems. Equations 16, 30 and 31 represent double acting cylinder and valve models. Equations 40-43 model single acting cylinders and their servovalves. Equation 38 can be used for design and selection process. The work is still going on.

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