HYDROSTATIC FORCE SENSOR

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ABSTRACT

This paper presents a theoretical and experimental investigation of a new kind of force sensor which detects forces by measuring an induced pressure change in a material of large Poisson's ratio. In this investigation we develop mathematical expressions for the sensor's sensitivity and bandwidth, and show that its sensitivity can be much larger and its bandwidth is usually smaller than those of existing strain-gage-type sensors. This force sensor is well-suited for measuring large but slowly varying forces. It can be installed in a space smaller than that required by existing sensors.

ARCHITECTURE

The objective is to design and construct a prototype hydrostatic force sensor to measure the compression and tension forces in one direction. Figure 1 shows a schematic of the sensor configuration. It consists of two components (part A and part B) that behave like a piston and a cylinder. The pressure in the fluid trapped between A and B is measured by a pressure transducer. A standard face seal prevents fluid leakage. Because this force sensor must measure both compression and tension, it is necessary to clamp part A and part B together with screws. The clamping force which we call *pre-load*, f_i, is applied by tightening the screws. The more the screws are tightened, the greater pre-load force that can be generated in the fluid. The force to be measured is f. If load f is a tension force, the fluid pressure decreases from the initial pre-load value. If load f is a compression force, the fluid pressure increases.

DESIGN CONSTRAINTS

Because the force sensor must measure both compression and tension, it is necessary to clamp part A and part B together with screws. The clamping force which we call *pre-load* f_i is applied by tightening the screws. If load f is a tension force, the screw force increases and the fluid pressure decreases. If load f is a compression force, the screw force decreases and the fluid pressure increases. The screws and the fluid chamber act like two springs in parallel. The stress in the screws and the pressure in the fluid can be calculated from equations 1 and 2.

$$\sigma_s = \frac{f_i}{n\,A_s} + \left(\frac{K_s}{K_s + K_f} \right) \frac{f}{n\,A_s} \qquad (1)$$

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$$p = -\frac{f_i}{A_f} + \left(\frac{K_f}{K_s + K_f}\right)\frac{f}{A_f} \qquad (2)$$

K_f and K_s are effective stiffness of the fluid and the screws and can be calculated from equations 3 and 4.

$$K_f = \frac{B\,A_f}{h} \qquad (3)$$

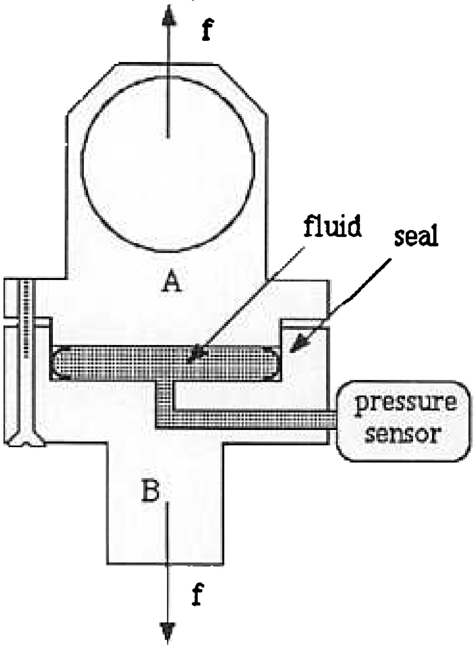
$$K_s = \frac{n\,A_s\,E_s}{L_s} \qquad (4)$$

A_s, E_s, L_s and n are the area, Young modulus, effective length and quantity of the screws, respectively. B, A_f, and h are the bulk modulus, fluid surface area, and fluid height, respectively. It can be observed from equations 1 and 2 that when f =0 (i.e., there is no force on the force sensor), the forces in the fluid and the screws are both equal to f_i. When force f is applied as a tension force (as shown in Figure 2), the stress in the screw, σ_s, increases while the pressure in the fluid decreases. If f is a tension force, there are two limiting situations that cause failure in the system:

Case 1: The stress in the screw, σ_s, reaches the material yield stress.
Case 2: The pressure in the fluid, p, decreases to zero.

If f is a compression force, two other limiting situations can cause failure in the system:

Case 3: The pressure in the fluid, p, reaches the maximum measurable pressure of the pressure transducer or the maximum allowable pressure of the seal.
Case 4: The screw compression stress reaches its maximum compression yield stress.



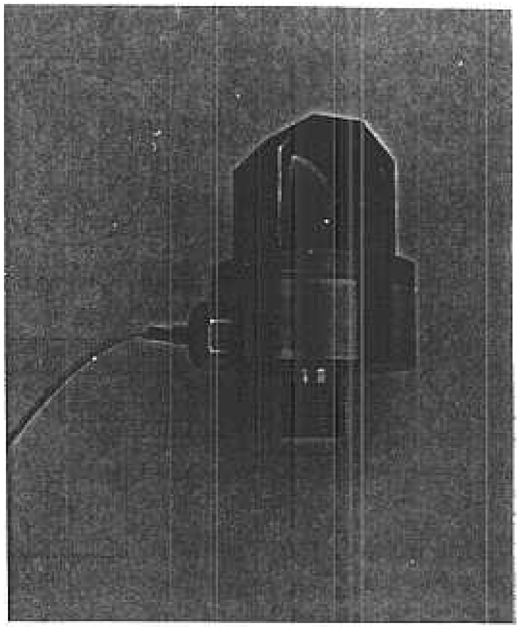


Figure 1: The pressure increase in a high Poisson ratio material is a direct result of applied force. If load f is a tension force, the fluid pressure decreases from the initial pre-load value. If load f is a compression force, the fluid pressure increases.

The designers must assure that the four limiting situations above never occur during normal system operation. Next we explain how to guarantee that these four cases do not occur.

Cases 1 and 2:

To guarantee that case 1 for a tension force (i.e., failure of the screw material) does not occur, the designer must ensure that the screw stress, σ_s, remains below the maximum allowable screw stress, σ_{all}, when f_{max} is imposed on the system. This is shown in inequality 5, where it can be observed that choosing a small pre-load, f_i, helps the designers keep σ_s smaller than σ_{all}. On the other hand, to guarantee that case 2 does not occur (i.e., the fluid pressure becomes zero), f_i should be chosen to be large enough to satisfy inequality 6.

$$\sigma_s = \frac{f_i}{n A_s} + \left(\frac{K_s}{K_s + K_f} \right) \frac{f_{max}}{n A_s} < \sigma_{all} \tag{5}$$

$$p = -\frac{f_i}{A_f} + \left(\frac{K_f}{K_s + K_f} \right) \frac{f_{max}}{A_f} < 0 \tag{6}$$

Solving for f_i from inequalities 5 and 6 results in an upper bound and a lower bound for the pre-load force, f_i.

$$\left(\frac{K_f}{K_s + K_f} \right) f_{max} < f_i < n\, \sigma_{all}\, A_s - \left(\frac{K_s}{K_s + K_f} \right) f_{max} \tag{7}$$

Inequality 7 is a design constraint which is necessary to prevent force sensor failure in the presence of the maximum tension force, f_{max}.

Cases 3 and 4:

To guarantee that case 3 for a compression force (i.e., excessive fluid pressure) does not occur, the designer must ensure that the fluid pressure, p, does not reach the maximum allowable pressure of the pressure transducer, σ_p, when f_{min} is imposed on the system[2]. (Note that f_{min} is a negative quantity; for the prototype force sensor f_{min} = -7,200 lb.) As seen in inequality 8, this can be ensured by choosing a small pre-load, f_i. On the other hand, to guarantee that case 4 (i.e., failure of the screw material under compression) does not occur, the preload force, f_i, must be chosen to be a large quantity, as seen in inequality 9.

$$-\sigma_p < -\frac{f_i}{A_f} + \left(\frac{K_f}{K_s + K_f} \right) \frac{f_{min}}{A_f} \tag{8}$$

$$-\sigma_{all} < \frac{f_i}{n A_s} + \left(\frac{K_s}{K_s + K_f} \right) \frac{f_{min}}{n A_s} \tag{9}$$

Solving for f_i from inequalities 8 and 9 results in an upper bound and a lower bound for the pre-load, f_i during compression.

$$-\left(\frac{K_s}{K_s + K_f} \right) f_{min} - n\, A_s\, \sigma_{all} < f_i < \sigma_p\, A_f + \left(\frac{K_f}{K_s + K_f} \right) f_{min} \tag{10}$$

Inequalities 7 and 10 must be satisfied to prevent sensor failure.

DESIGN PARAMETERS:

Two properties of this sensor, <u>sensitivity</u> and <u>bandwidth</u>, are its major parameters. The "sensitivity" of a sensor tells us the quality of the signal (i.e., its resolution in volt/lbf). The "bandwidth" of a sensor tells us the range of force signal speeds that this force sensor can measure. Force sensors can record only the frequency components of the applied forces which fall within the sensor's bandwidth. If the bandwidth of the sensor is not wide in comparison with the bandwidth of the rest of the system (e.g., robot, actuation, etc.), either the force

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2 σ_p must be chosen to be the smallest of the maximum measurable pressure of the pressure transducer or the maximum allowable pressure of the seal.

sensor dynamics must be modeled for controller design, or a lower bandwidth for overall control system should be considered.

The pressure in the fluid can be calculated via equation 6 and it is re-written here in a more appropriate form.

$$p = \frac{K_f}{K_s + K_f} \frac{1}{A_f} (f - f_i) \qquad (11)$$

Since the voltage from of the pressure transducer, v, is proportional to the pressure increase, equation 12 applies.

$$v = S_p \; p \qquad (12)$$

S_p is the pressure transducer sensitivity. Substituting equation 11 into 12 results in the output voltage as a function of the applied force.

$$v = \frac{K_f}{K_s + K_f} \frac{S_p}{A_f} (f - f_i) \qquad (13)$$

The force sensor sensitivity therefore equals:

$$S = \left(\frac{K_f}{K_s + K_f} \right) \frac{S_p}{A_f} \quad \text{volt/lbf} \qquad (14)$$

Designers always wish to have a large sensitivity in the sensor: a large sensitivity in the force sensor results in a large voltage for a given applied force. The parameters of equation 14 can be chosen to yield a particular sensitivity. On the other hand, the designer should be aware of the role of the design parameters on another important sensor property: bandwidth. The overall bandwidth of a robotic system is limited by high-frequency unmodeled dynamics (e.g., structural resonances for bending and torsion, sensor dynamics, actuator dynamics). To achieve a wide bandwidth for the closed-loop system, it is necessary to consider high order dynamics in modeling the system. Adding high order dynamics to the system results in a wider bandwidth for the system at the expense of a high order compensator. If higher order dynamics cannot be determined, it is necessary to compromise on the overall system bandwidth. It is usually recommended to "push" the high frequency unmodeled dynamics by designing "stiff" components. In other words, a robot's components must be designed to have large natural frequencies. The natural frequency or bandwidth of a sensor can be calculated from equation 15.

$$\omega \cong \left[\frac{K_t}{m} \right]^{1/2} \qquad (15)$$

K_t is the stiffness of the sensor and m is some effective mass that depends on the rest of the robot inertia. It is rather impractical to arrive at the natural frequency of the force sensor without any regard for the inertia of the other components. We leave equation 15 without further development, since m is a function of robot inertia. However, we must consider that the larger the stiffness of the sensor, the larger the natural frequency is. The total stiffness of the pressure transducer can be derived from equation 16.

$$K_t = K_s + K_f \qquad (16)$$

To achieve a large stiffness, both K_s and K_f must be large. From equation 14 it can be observed that a large sensitivity requires a small A_f, but equation 3 shows that a small A_f results in a low fluid stiffness. One method of dealing with this tradeoff is to decrease h, so K_f does not get too small. Another tradeoff is the screw stiffness: a large screw stiffness results in a large total stiffness of the system, but this decreases the system sensitivity. Equation 16 shows that stiff screws decrease the system sensitivity. We recommend that, for low bandwidth yet still precise operation, the designer choose a set of screws with small stiffness. On the other hand, in wide bandwidth operations, we recommend a large stiffness for the screws.